

## Book reviews

### **Inverse Problem Theory. Methods for Data Fitting and Model Parameter Estimation**

Albert Tarantola. *Elsevier, Amsterdam and New York, 1987, 613pp., ISBN 0444427651. US\$80/Dfl.180*

Inverse theory has applications in many branches of science, but it is particularly important in Geophysics because direct measurements cannot be made deep inside the Earth—we are forced to make what inferences we can from measurements made at the Earth's surface. There are still very few books on inverse theory, so this is a welcome addition. Books on the subject can be divided into 'how-to-do-it' manuals which concentrate on methods and their application to specific common problems, and 'theoretical' works which concentrate on the foundations of inverse theory and underlying philosophy. Menke's book on inverse theory falls into the former category, this book into the latter category.

Inverse theory can be developed in either a 'statistical' approach, in which it emerges as an extension of conventional parameter estimation, or an 'exact' approach, in which inversion of precise and complete data (perhaps involving an infinite number of observations) is studied first, the effects of incomplete but precise data are studied second, and errors in the observations are considered last of all. The early work of Gel'fand and Levitan considered only exact data and concentrated on finding an algorithm to solve a specific inverse problem; Backus & Gilbert's first papers concentrated on exact data. Proof of uniqueness and stable algorithms are of paramount importance in inversion, but they are specific to the particular problem at hand and it is impossible to present a general treatment. This book adopts the statistical approach; little effort is made in developing specific examples with complete and perfect data.

Inversion methods divide also into discrete and continuous formulations, depending on whether the physical quantity to be modelled is represented in terms of a finite set of model parameters, such as coefficients in a power series representation of a function, or a continuous function. In the continuous case the number of model parameters is infinite. The two approaches can be made to yield similar results, and the choice is largely a matter of taste. The advantage (some might say disadvantage!) of inverse theory over conventional parameter estimation is that it allows us to evaluate uncertainties arising when the measurements do not sample some aspect of our model. Through the concept of *resolution* it is possible to relate precisely the chosen estimate of a model to the 'true' model (but not *vice-versa*).

This book is divided into two parts, the first being on discrete inverse problems and the second on continuous problems, but the major emphasis is on the discrete case. Professor Tarantola justifies this emphasis: there are severe difficulties in generalizing the concept of a probability density function to infinite dimensional vector spaces, and

furthermore a continuous inverse problem can be discretized to any desired degree of accuracy to yield a discrete inverse problem, provided we can solve, in principle at least, the mathematical problem posed by perfect data.

Part I of the book begins with an introduction to discrete inverse problems. The main statistical results, such as Bayes' theorem and Shannon's concept of information, are introduced at this stage. This is followed by two short chapters on 'direct' or 'forward' methods—trial and error and Monte Carlo. There follows a long chapter on the least-squares criterion of fit. These are by far the most common methods in use in geophysics. However, least-squares methods do not perform well when the data contain outliers, or the error distribution has a long tail. In such cases minimizing the  $l_1$ -norm rather than the  $l_2$ -norm can produce better results. Chapter 5 introduces the general  $l_p$ -norm criterion, and develops linear programming ( $p = 1$ ) and minimax ( $p = \infty$ ) methods in considerable detail. This part of the book is likely to be the most influential in converting geophysicists to new methods. The mathematical style of this, the easier discrete part of the book, is difficult but sound. Unfortunately the examples are rather pathetic. Even the geophysical examples lack any real interest for me: the easy ones are rather trivial and have not been developed sufficiently to illustrate the general theory, while the difficult ones involve little more than substituting into general formulae.

Part II deals with continuous inverse problems solved by least-squares. It contains a thorough background to the functional analysis needed to understand the theory, and is rather easier to follow than most of the original papers. The method of Backus and Gilbert is given a full description. Again I found the examples poor: they are used to illustrate the power of inverse theory rather than to aid understanding of the general problem. The beginner in geophysics will find it very hard to relate the specific problems of seismic tomography or waveform inversion to the background theory.

This is a long book, containing a great deal of basic theory and useful methods and algorithms. The mathematics is sound but the presentation has been let down by poor examples and bad printing (particularly the grey boxes, which are very fuzzy). The 'statistical' approach to inverse theory given in this book is more useful in practice than the 'exact' approach because it leads directly to useful algorithms; I would therefore expect it to become a common reference work. The difficult mathematical style and lack of really good examples make it unsuitable as a course book in either geophysics or statistics; the student in geophysics would be better advised to start with a simpler book (like Menke) or Parker's review papers.

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