Deconvolution of marine seismic data using the $l_1$ norm

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Summary. The trace measured in a marine seismic experiment can be expressed as the convolution of a source wavelet and a sparsely populated spike train which represents the impulse response of the medium. Deconvolution processing using the $l_1$ norm criterion has been suggested by Taylor, Banks & McCoy to obtain estimates of the impulse response from noisy traces. This criterion is well-suited to problems where a spiky output is expected. In this paper we present a new $l_1$ formulation of the deconvolution problem which represents a significant improvement over the concept proposed by Taylor et al. The performance has been tested with synthetic data where noise has been added both to the trace and the source wavelet, and the results of these trials are described. An example is presented which demonstrates the success of this method in removing bubble pulse interference from a bottom-reflected acoustic signal.

1 Introduction
Bottom-reflected acoustic signals obtained in marine seismic experiments can be used to determine the structure of ocean bottom sediments. In particular, we have carried out experiments with small explosive charges to probe the surficial sediment layers. The bottom-reflected signals measured in these experiments can be represented as the convolution of a noisy wavelet, representing the acoustic source, and a sparsely populated spike train, representing the impulse response of the medium. This relationship is conveniently expressed as

$$t = w \ast s + r$$

where $t$ is the signal trace, $w$ the source wavelet, $s$ the spike train and $\ast$ denotes convolution. The noise $r$ is assumed to be additive and of different statistical character than either $w$ or $s$. Thus, if $w$ is of length $K$ and $s$ is of length $M$, the trace $t$ is of length $N = M + K - 1$.

The general problem in processing the data is to deconvolve the impulse response from the measured trace, and hence deduce a layered structure of the sediment. This problem is made difficult by the presence of bubble pulse signals from the explosive sound sources used.
in the experiments. An example of the waveform of a 1.8 lb SUS (Signal Underwater Sound) charge showing the bubble pulses following the initial shock wave is presented in Fig. 1. The influence of the bubble pulses is demonstrated in the same figure where a typical trace from a bottom reflected signal is plotted below the source waveform. After about 20 ms following the arrival of the first reflection from the seafloor, the trace is severely contaminated by bubble pulse signals which may be masking later reflections from deeper layers.

Several methods of deconvolving the impulse response using various schemes such as the $l_2$ norm (Santaniello et al. 1979; Robinson & Treitel 1980; Dicus 1981), the varimax method (Ooe & Ulrych 1979), and the $l_1$ norm (Taylor, Banks & McCoy 1979), have been proposed and have been applied to data collected in seismic experiments. In this paper we outline a new approach to deconvolution using the $l_1$ norm, and illustrate the method by numerical results obtained with both simulated traces and real data.

![Source Waveform](image)

![Bottom Reflection](image)

**Figure 1.** The source waveform shown is a typical signal from an underwater explosive charge of 1.8 lb at 200 m depth. The lower trace is a bottom reflection from an explosive charge over an abyssal plain.
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Our general approach in deconvolving the impulse response is to seek an estimate of the spike train which minimizes the expression $t - w * s$ in (1). Assuming that $w$ is known in advance, this expression can be written in matrix notation as

$$-Ws = r$$

(2)

where the elements of the $N \times M$ matrix $W$ are defined by $W_{ij} = w_{i-j+1}$, for $1 < i - j + 1 < K$, and $W_{ij} = 0$ otherwise. The matrix equation (2) can be 'solved' for $s$ by minimizing $\| r \|$, i.e. by minimizing $\| t - Ws \|$. (In marine seismic work it is reasonable to assume that initial

Figure 2. Results of deconvolution of a noisy simulated trace ($S/N \approx 5$). The 8-element spike train $\hat{s}$ is an estimation of the original spike series.
Figure 3. (a) The 15-element spike train $\hat{s}$ represents an estimation of the sediment impulse response. (b) The spike train $\hat{s}$ represents an estimation of the sediment impulse response after 30 spikes have been extracted. The convolution of $w * \hat{s}$ closely resembles the original trace.

We have used the $l_1$ norm $\| \cdot \|_1$ defined by

$$\| r \|_1 = \sum_{i=1}^{N} |r_i|$$

when solving (2). As was previously pointed out (Claerbout & Muir 1973), this norm provides a more robust procedure than the usual least-squares criterion when handling
certain types of errors (e.g. erratic data) and noise distributions (e.g. non-Gaussian) that occur in geophysical applications. Before describing our own approach, let us briefly review an earlier $l_1$ deconvolution algorithm.

### 2.1 The Method of Taylor et al.

A deconvolution scheme using the $l_1$ criterion was proposed by Taylor et al. (1979). The formulation suggested by them was to minimize the expression

$$\sum_{i=1}^{N} |r_i| + \lambda \sum_{j=1}^{M} |s_j|$$

(3)
where $\lambda$ is an adjustable parameter. For large values of $\lambda (~100)$ the output spike train is zero, and as $\lambda$ is decreased the number of spikes obtained increases. This particular formulation was chosen in analogy with the familiar $l_2$ case where the expression to be minimized is

$$
\sum_{i=1}^{N} |r_i|^2 + \lambda \sum_{j=1}^{M} |s_j|^2.
$$

(4)

In expression (4) the second term is a stabilizing or prewhitening condition, and the parameter $\lambda$ is interpreted as the level of prewhitening additive noise. (This expression contrasts, for example, with that of a shaping filter method due to Treitel & Robinson 1966, appendix II.) In the case of (3) Taylor et al. (1979) concluded from their simulation study that an 'optimal' value usually exists for this parameter within the range $15 < \lambda < 40$; this was determined by varying $\lambda$ and comparing the trace with the result of convolving $w$ with the extracted spike train. There is no guarantee, however, that a value of $\lambda$ chosen within these bounds will be suitable for other traces, and in practice the choice of an 'optimal' $\lambda$ involves considerable computation and experimentation.

2.2 A NEW $l_1$ METHOD

In this section a new method of deconvolution with the $l_1$ norm is outlined which uses a simpler strategy to extract a spike train. The problem is posed as in equation (2), i.e. we seek an estimate $\hat{s}$ of the spike train $s$ from the trace $t$ given some initial information about the source wavelet $w$. In our approach the quantity $\| t - Ws \|_1$ is partially minimized, using linear programming techniques, subject to the condition that a specific number of spikes will be extracted. This formulation entirely eliminates the parameter $\lambda$. Our algorithm constructs $\hat{s}$ in a manner that causes $\| t - Ws \|_1$ to decrease by the maximum amount possible as each spike is inserted in an optimal position. Consequently, relatively few spikes are usually sufficient to cause a significant decrease in $\| t - Ws \|_1$.

This method is well-suited to problems where the desired number of spikes in the impulse response is known approximately in advance. In other cases the method is still effective, since the number of spikes necessary to provide an accurate representation of the impulse response can be determined automatically by monitoring the decrease of the objective function

$$
\sum_{i=1}^{N} |r_i|
$$

as the number of spikes extracted is successively increased.

3 $l_1$ deconvolution of simulated data

A simulation study with an arbitrary spike series and a wavelet designed to represent the multiple pulse signature from an explosive charge was carried out to determine the performance of the method in deconvolving noisy signals. Gaussian noise was added to either the wavelet or the convolved trace, or to both waveforms to achieve various conditions of signal-to-noise ratio. An example from this study of spike train deconvolution is presented in Fig. 2. Noise has been added to both the wavelet $w$ and the convolved trace $t$ to achieve a signal-to-noise ratio of 5. (This ratio is smaller than the signal-to-noise conditions commonly measured in our experiments.) The extracted spike series, $\hat{s}$, for eight spikes shown at the
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bottom of the figure demonstrates that our method is successful in deconvolving the original spike train from a relatively severe condition of noise in the trace.

4 $l_1$ deconvolution of the bottom reflected signals

The method has been applied to the data sample shown in Fig. 1, a time series segment containing the bottom bounce reflection from a 1.8 lb charge exploded over an abyssal plain site in the north-east Pacific. In this example, an estimate of the wavelet was obtained from the direct water-borne arrival at the hydrophone. The performance of the method in deconvolving the ocean bottom impulse response is demonstrated in the last two figures. The deconvolved spike train after the extraction of 15 and 30 spikes is shown below the original reflection series in Fig. 3(a, b) respectively. The bubble pulse interference after about 20 ms following the arrival of the first reflection has been greatly suppressed, and the convolved series $w \ast \hat{s}$, plotted at the bottom of the figures, closely resembles the original data even after only 15 spikes have been deconvolved. The objective function

$$\sum_{i=1}^{N} |r_i|$$

is plotted in Fig. 4 to show the convergence of the $l_1$ curve fit as the number of extracted spikes is increased from 1 to 35. Little significant decrease in the objective function occurs after about 30 spikes, by which time the most recent spikes are of small amplitude. Hence, further spike extractions are not justified, and so the 30-element spike series shown in Fig. 3(b) is taken to be a satisfactory estimate of the impulse response.

5 Remarks

This initial study has demonstrated the feasibility of our approach, and the method is now being used to analyse bottom reflectivity data to determine the sediment structure of a site.
in the north-east Pacific. The possibility of extending our method to reflectivity data where the source wavelet is not accurately known is presently under active investigation.

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References