The Earth's rotation and atmospheric circulation, from 1963 to 1973

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'If everybody minded their own business, the world would go round a deal faster than it does.'

Alice's Adventures in Wonderland

Lewis Carroll

Summary. The zonal angular momentum of the atmospheric circulation has been evaluated month-by-month and compared with astronomical observations of the length-of-day for the 10 years from 1963 May to 1973 April. The reason for undertaking this study is to enable the astronomical observations to be 'corrected' for the zonal wind effect and to investigate the residual excitation function for solid-Earth contributions. The principal conclusions reached are the following: (i) The annual change in length-of-day is almost entirely due to the seasonal changes in the zonal circulation with tidal, oceanographic and hydrologic phenomena contributing together at most 10 per cent of the total excitation. (ii) The semi-annual term is predominantly due to the zonal wind and the body tide, with oceanic and hydrologic terms contributing about 10 per cent. (iii) The atmospheric circulation plays a dominant role in length-of-day changes in the period range from 1 to about 4 yr. This is partly associated with the quasi-biennial oscillation and its harmonics. Both the period and amplitude of these fluctuations are very variable. (iv) At longer periods the atmosphere may still contribute to the total excitation but other excitation functions begin to rise above the spectrum of the meteorological excitation. (v) At periods less than about 1 yr the atmospheric excitation is dominant, and while the presence of other excitation functions cannot be excluded, they cannot exceed 20 per cent of the wind excitation. On the basis of these results the astronomical record from 1962 to 1978 has been 'corrected' for the meteorological 'noise'. The residual excitation exhibits only fluctuations on a time-scale of about 5 yr and longer and it is this result that must be attributed to core–mantle interactions or to other solid-Earth excitation functions.

1 Introduction

In a series of papers, Lambeck & Cazenave (1973, 1974, 1976, 1977) (see also Lambeck 1980) discussed the interaction between the zonal wind circulation and changes in the Earth's rotational velocity or the length-of-day (lod). The discussion in the first paper was limited to the seasonal variations in lod, including a quasi-biennial oscillation. A satisfactory quantitative agreement was found between the astronomically observed lod fluctuations and the meteorological estimate of changes in the zonal angular momentum component of the atmosphere at the semi-annual and annual frequencies, confirming earlier but more qualitative work by Munk and co-authors (see Munk & MacDonald 1960) that the zonal winds are an important factor in lod changes. In Paper I (Lambeck & Cazenave 1973) a good quantitative agreement between cause and effect was also found at the biennial frequency, confirming an earlier qualitative comparison by Iijima & Okazaki (1966). In particular it was noted in Paper I that the irregular behaviour seen in lod data was mainly a consequence of the variable intensity and frequency of the quasi-biennial zonal atmospheric circulation. Predictions of these variations, based on the lod data, were qualitatively verified in the second paper in which the exchange of angular momentum at higher frequencies, from 2 to 6 cycles per year (cpy), was mainly discussed. In the third paper, possible meteorological contributions to the long period (> 2–3 yr) variations in the length-of-day were considered; a possibility also investigated by Siderenkov (1969). The fourth paper discussed *inter alia* the year-to-year variability seen in the seasonal terms of lod and speculated on the likely changes in the atmospheric circulation that are required to explain this astronomical observation.

These studies demonstrated that an exchange of angular momentum between the atmosphere and solid Earth occurs over a wide range of periods and that this may mask other geophysical perturbations in the Earth's rotation. If so, this stresses the need to know the atmospheric net zonal angular momentum if other geophysical excitations are to rise above this 'meteorological noise' (see also Hide 1977 and Lambeck 1980). On the other hand, these studies may well provide some insight into the variability of the global atmospheric circulation. Some more or less definitive conclusions drawn in these studies by Lambeck & Cazenave were that:

(i) The annual and semi-annual changes in lod are – apart from a strong tidal contribution to the semi-annual term[*] – almost entirely due to the zonal atmospheric circulation.

(ii) Considerable fluctuations in these terms have been observed in the astronomical data during the last 20–25 yr, not inconsistent with the observation that there has been an overall decrease in the strength of the zonal circulation during this time interval.

(iii) The evidence for the biennial term in the lod indicates that this term is present down to considerable depths in the atmosphere, and the variability of the amplitude and period of the lod term may provide some constraint on the global aspects of this atmospheric circulation pattern.

(iv) At higher frequencies, from 2 to 6 cpy, the variable changes in lod are almost entirely a consequence of an irregular zonal circulation in all parts of the atmosphere but with a significant contribution coming from the high latitude circulation reversals in the winter hemispheres.

(v) There may be considerable variability in the net zonal winds at frequencies above

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[*] While a glance through Sir Harold Jeffreys' collected works reveals that aspects of the Earth's rotation have been of interest to him for much of his career, he has written little on the fluctuations in lod. In an early paper (Jeffreys 1928) he predicted the presence of tidal effects in lod, long before they were actually detected.

6 cpy, particularly about the tidal frequencies of about 13 and 27 cpy but also at frequencies as high as 30–40 cpy.

(vi) At lower frequencies, < 0.5 cpy, the atmospheric circulation may contribute to the total excitation function, possibly by 10–20 per cent at the frequencies corresponding to the so-called 'decade' fluctuations.

These conclusions were based mainly on a comparison of astronomical observations with zonal wind data for a 5-yr period, 1958 April to 1963 May, and with atmospheric pressure indices for a longer period. Both the wind and lod data for this 5-yr period have their difficulties. The astronomical data suffered from a change in the reduction procedures in 1962 January and Okazaki (1977) has pointed out that this may result in an erroneous estimate of the semi-annual term from observations made prior to this date. The wind data used was mainly confined to latitudes between ± 45° and while this appeared satisfactory for the seasonal terms, much of the month-to-month variability in rotation is also due to the higher latitude winds.

Additional wind data for the 10-yr period 1963–73 is now available that generally covers a greater latitude range than did the earlier set. This has enabled a number of the above-mentioned problems associated with the exchange of angular momentum between the Earth and atmosphere to be re-examined, the results of which are discussed in this paper.

2 Equations of motion

The zonal component of the angular momentum of the atmosphere relative to Earth-fixed axes is (Lambeck 1980)

$$h \equiv \int_{r=R_\phi}^{\infty} \int_{\phi=-\pi/2}^{\pi/2} \int_{\lambda=0}^{2\pi} \rho(r) r^3 \cos^2 \phi \, u_\lambda(r, \phi, \lambda) \, dr \, d\phi \, d\lambda, \tag{1}$$

where u_λ is the zonal wind component at geocentric distance r, latitude ϕ and longitude λ. $\rho(r)$ is the density of the atmosphere. Height is measured with respect to R_ϕ, the Earth's radius given by an ellipsoidal approximation. The proportional change in lod is equal to $h/\Omega I$, where Ω is the mean angular velocity and I the polar moment of inertia of the Earth. The negative of this quantity

$$\psi = -h/\Omega I, \tag{2}$$

is referred to as the excitation function. Astronomers do not directly observe the instantaneous angular velocity, ω, of the Earth. Rather, they measure the integrated amount by which the planet is slow or fast over a number of days, relative to a uniform time-scale established, since 1955, by atomic clocks and denoted by AT. This gives an estimate of

$$\tau = -(\text{UT1} - \text{AT}),$$

where UT1 is the Universal Time (corrected for polar motion) kept by the Earth. The quantity of interest is

$$m \equiv \frac{-\Delta(\text{lod})}{\text{lod}} = \frac{\omega - \Omega}{\Omega} = -\frac{d\tau}{dt} = \frac{d}{dt}(\text{UT} - \text{AT}), \tag{3}$$

and represents the proportional change in lod which is a measure of the variations in angular velocity about the instantaneous rotation axis. The equation of motion is

$$m = \psi. \tag{4}$$

The excitation function ψ represents the total of all geophysical mechanisms perturbing the Earth's rate of rotation and would include phenomena that result in (i) mass redistribution in the solid Earth, oceans and atmosphere, (ii) changes in angular momenta due to motions in the core and hydrosphere relative to axes fixed in the planet, and (iii) external torques acting on the body. The geophysical problem is to evaluate the excitation function taking all these factors into account. The astronomical problem is to observe the changes in rotation (see Lambeck 1980). We will consider mainly the atmospheric excitation function in this paper. This consists of two parts, that due to the winds (equation 1) and that due to changes in the inertia tensor or mass distribution within the atmosphere. In problems associated with changes in the direction of the rotation axis this latter contribution is the dominant one, but in lod problems it is virtually negligible. An upper limit to the contribution of the atmospheric circulation to lod changes is obtained by considering the angular momentum of the global atmosphere. This varies from $1.0 \times 10^{33}\,\mathrm{g\,cm^2\,s^{-1}}$ during the northern hemisphere summer to about $1.5 \times 10^{33}\,\mathrm{g\,cm^2\,s^{-1}}$ during the winter, so if the atmosphere ceased rotating instantaneously, the excitation function would change by about 2 to 3 parts in 10^8 and the lod would be changed by about 1½ to 2 ms by virtue of equation (2). This represents an upper limit to wind-induced changes in lod.

In the absence of excitations other than the zonal wind, the equation of motion (4), with (2), is valid only if (i) there is no elastic yielding of the mantle by the wind torques at the surface of the Earth, (ii) the influence of the wind on the ocean circulation is negligible, and (iii) the core is coupled to the mantle's variable rotation. In terms of a longitudinally averaged zonal wind $\bar{u}_\lambda$, the surface stress is

$$\bar{\tau}_{r\lambda} = -D\rho\bar{u}_\lambda(R_\phi, t)\bar{u}_\lambda(R_\phi, t),$$

and the excitation function is (Lambeck 1980)

$$\psi(t) = \frac{-2\pi R^3}{I} \int_{t=0}^{t} \int_{\phi} \bar{\tau}_{r\lambda} \cos^2\phi\, d\phi\, dt, \tag{5}$$

where D is an average friction coefficient, reflecting surface and mountain stresses. Its value can be obtained by equating the angular momentum integral (1) with the surface torque integral (5). Only the second-degree zonal harmonic component in $\bar{\tau}_{r\lambda}$ will modify the polar moment of inertia of the Earth but, because of the asymmetrical distribution of the annual wind cycle with respect to the equator, this component is quite small. Once this component is known, the change in inertia tensor of the solid Earth follows from the shear Love numbers, defined analogously to the load Love numbers. The final result is $\Delta I/I \simeq O(10^{-12})$ and the effect is negligible compared with the other excitations (Section 5) that are of the order 10^{-10} or greater.

The ocean response to the zonal wind is twofold. One is that the wind induced currents contribute to the relative angular momenta of the Earth and this is discussed further in Section 5; the other is that the moment of inertia may be modified by the piling up of water at north—south continental barriers. For the equatorial asymmetric annual winds the effect on the inertia tensor will be small but it may be more significant at the semi-annual period. The presently available ocean surface wind data is not sufficient to investigate this contribution further. The core coupling effect is more important, for if the core does not participate in these changes at the seasonal frequencies, ψ as defined by equation (2) will be increased in the ratio I/I_{mantle} or by a factor of about 1.1. The absence of effective mechanisms to couple the Chandler motion to the core (Rochester 1970; Lambeck 1980) also makes it improbable that the core partakes in the seasonal and higher period fluctuations of the Earth's mantle.

3 Astronomical data

The precise astronomical observations of lod since the introduction of atomic time in 1955 have been discussed by Lambeck (1980). Prior to 1955 the Earth's rotation was monitored with respect to time standards kept by pendulum and crystal clocks and in 1936 N. Stoyko observed for the first time seasonal variations in lod. Later, in 1953, H. Smith and R. Tucker reported the existence of a semi-annual term, part of which could be attributed to the solid tides as predicted by Jeffreys in 1928. Tidal terms at 14 and 27 days were not firmly recorded until the atomic time-scale was established. Despite the introduction of atomic time, the astronomical observations since 1955 are still of variable quality although the data since 1962 are fairly uniform in that they have been reduced to a self-consistent system. We have chosen the smoothed 5-day UT1−AT values given in the annual reports of the Bureau International de l'Heure (BIH) since this is the only uniform data set from 1962 to the present. The smoothing that has been applied by the BIH suppresses most information at frequencies above about 12 cpy but this is of no consequence in the present study where only monthly mean excitation functions have been established. The precision of the 5-day means is of the order 1.5–2.0 ms for the earlier years 1962–68 and 1.0–1.5 ms for the latter period with successive values being strongly correlated as a result of the smoothing used by the BIH. As indicated by equation (4) the quantity of geophysical interest is not the $\tau \equiv -(UT1-AT)$ but its time derivatives, m and $\dot{m}$. These have been estimated by fitting a spline function through τ using the above accuracy estimates. Fig. 1 illustrates the results of the 17-yr period from 1962 to 1979 in which all data have been reduced in a homogeneous manner.

If σ_τ^2 is the variance of τ, the error spectrum of m is

$$E_m(f) = (2\pi f)^2 E_\tau(f) \qquad s, \tag{6}$$

where

$$E_\tau(f) = \sigma_\tau^2/f_N \qquad s^{-1}$$

is the power spectrum of the observational errors in τ for uncorrelated observations. f_N is the Nyquist folding frequency. Integrating equation (6) from zero frequency to f_N gives the variance of m or

$$\sigma_m^2 = \frac{1}{3}(2\pi f_N)^2 \sigma_\tau^2. \tag{7}$$

Because of the *ad hoc* smoothing applied to the BIH data the white noise model for σ_τ^2 is not appropriate, but as the zonal wind excitation functions can only be evaluated as monthly mean values, we adopt $\sigma_\tau \simeq 1.0$ ms for monthly mean values of τ. Then with $f_N = (2 \text{ month})^{-1}$

$$E_m(f) = 5.2(2\pi f)^2 \qquad s \tag{8a}$$

and

$$\sigma_m^2 = 4.8 \times 10^{-19}. \tag{8b}$$

These simple error spectra and variance estimates do not take into account the finite length L of the data set nor the discrete sampling interval ΔT, but as $\Delta T \ll L$ they will be adequate at frequencies considerably greater than L^{-1}.

The periodic annual and semi-annual changes are readily evident in m as are some longer term variations. These long-term fluctuations are generally attributed to geophysical excitation functions of other than meteorological cause and, in the first instance, they do not concern us here. The change in m for 1962 to about 1972, for example, is about 2×10^{-8} and near the upper limit of lod changes that can be attributed to the zonal wind circulation.
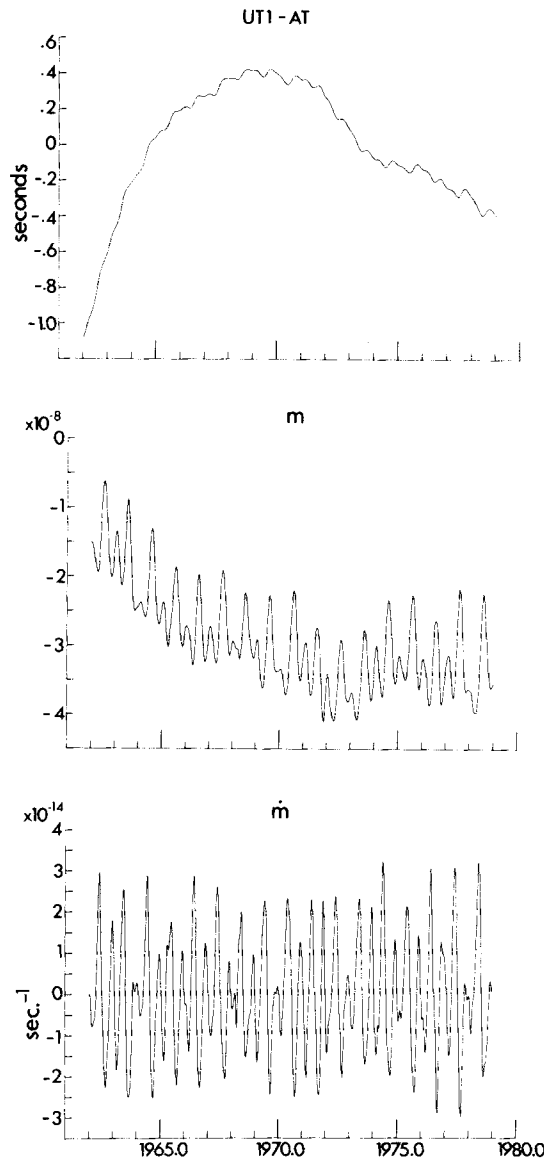
Figure 1. The astronomical evidence for changes in the Earth's rotation rate from 1962 to 1978. (a) $\tau = -(UT1 - AT)$; the integrated amount by which the Earth is slow or fast after a least squares removal of a linear term. (b) $m = -d\tau/dt$; the proportional change in the length-of-day using the data in (a) before the removal of the linear term. (c) dm/dt.

The seasonal terms are almost entirely of meteorological origin but in between these two period ranges both meteorological and 'solid-Earth' excitations may contribute to the perturbations in rotation.

4 Zonal wind excitation function

The wind data used in this study has been obtained from Environmental Research and Technology, Inc., (ERT) and part of the northern hemispheric data has been previously

discussed by Rosen, Wu & Peixoto (1976). The original data consist of daily or twice daily wind speed values measured mainly by radiosonde and radiowind techniques. Time averages have been estimated to give monthly mean zonal wind velocities for each station at a number of constant pressure levels, from the ground to 10 mb. It is these monthly station values that were obtained from ERT. For the years 1963–68, typically some 1000 stations reported wind data at altitude increments ranging from 1.2 to 4.6 km. The southern hemispheric coverage is variable but generally extends down to at least 55° S, except that from 1966 May to October the coverage extended only to about 40° S. For the years 1968–73 the average number of stations increased to about 1250 and the altitude increments ranged from 0.4 to 4.6 km. Fig. 2 illustrates the distribution of stations that reported wind measurements at altitudes up to 10 mb in a typical month during the second 5-yr interval. The coverage is generally good for the northern hemisphere as well as for southern latitudes down to about 40°–45°. Beyond that, the data are sparse but the monthly zonal winds for any one station also appear to be quite representative of that latitude generally. Presumably this is a consequence of the absence of large land masses in these southern latitudes so that the disturbances which appear on monthly mean maps, for example, are small compared with the mean zonal flow.

In order to evaluate equation (1) it is necessary to interpolate the available monthly zonal wind velocity data on to a regular four-dimensional grid (r, ϕ, λ, t). However, as well as being computationally expensive, the paucity of data in some regions makes the procedure impractical. Instead, the zonally averaged wind values $\bar{u}_\lambda$ were estimated for each month at each height at which data was given, by averaging all values in 5° latitude bands. Each of these averages is therefore associated with a particular height and an average latitude being the mean of the station latitudes which yielded data at the particular height and within the 5° latitude band. These $\bar{u}_\lambda(r, \phi)$ values were then interpolated on to a regular grid in height and latitude by fitting a quadratic surface through data values in the neighbourhood of the required grid points. At the poles the zonal wind speed was set to zero.

The estimate to the relative angular momentum follows from equation (1) as

$$h = 2\pi \int_{r=R_\phi}^{R_\phi + H} \int_{\phi=-\pi/2}^{\pi/2} \rho(r) r^3 \cos^2 \phi \, \bar{u}_\lambda(r, \phi) \, d\phi \, dr, \tag{9}$$
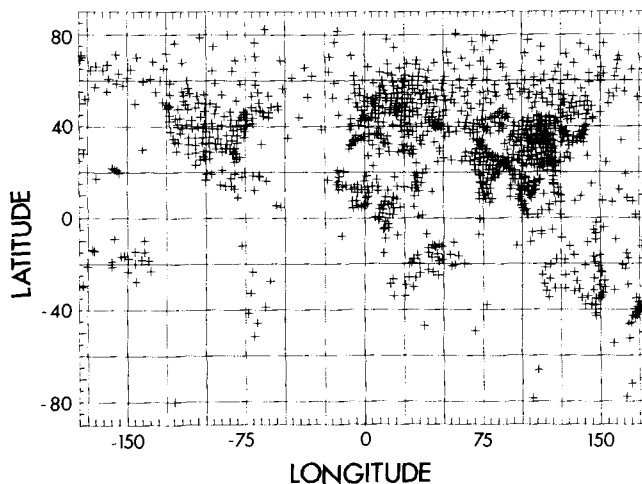

Figure 2. Distribution of stations that reported at least one daily zonal wind observation at altitudes from the ground to the 10 mb level during 1968 December. This distribution is typical for the years 1968–73.

where H is the altitude of the upper level at which wind speed observations were available, in this case the 10 mb pressure level or approximately 31 km. Winds above this altitude do not contribute significantly to the excitation function except at the 6- and 12-month periods (see below).

The 10 yr of data, from 1963 May to 1973 April were analysed in this manner. Fig. 3 illustrates the average zonal profiles for 1968–73 for the months from December to June. These results are very similar to those given by Newell *et al.* (1974) for the years 1958–63. The December and June profiles clearly exhibit the strong westerly winds in the two hemispheres flanking an equatorial region of easterly winds with generally low velocities. The cores of the westerly circulation show a pronounced seasonal pattern, with the winter hemisphere core being both more intense and closer to the equator than the summer hemisphere core. The high altitude polar winter jet is more pronounced in the southern hemisphere than in the northern hemisphere. The easterly winds cover nearly half of the latitude band but their speeds are low compared with the westerly winds. Results of the average zonal profiles for February, March and April indicate both a change in the latitudinal extent of the easterlies and in the strength of the circulation, changes that follow a semi-annual pattern.

Fig. 4 illustrates some latitude-band mean zonal wind data for the 10-yr period. The equatorial profile shows a succession of easterly and westerly winds propagating downwards in the lower stratosphere with roughly a 2-yr periodicity. This is the quasi-biennial oscillation. This profile illustrates clearly the variability in the period of the oscillation, with the interval between successive easterly cycles being longer during 1964–69 than for the years 1969–73. Such a behaviour had earlier been deduced from the astronomical data (Paper I) and was also suggested by the zonal wind observations at the Canal Zone (Wallace 1973). Below the tropopause the circulation has little obvious structure but, due to the higher atmospheric density, any residue of this cycle can contribute quite significantly to the excitation function. Away from the equator, the biennial oscillation pattern quickly vanishes, particularly in the northern hemisphere. The circulation at 20° north latitude has evolved into two oscillations, one centred in the troposphere and the second in the lower stratosphere. Vestiges of the biennial oscillation can still be seen in the lower stratosphere where the intensity of the easterly maximum is generally greatest in January of even years. At 20° S a similar circulation pattern is seen except that a semi-annual pattern is also evident in the upper troposphere. At latitudes ±40° the annual cycle is quite regular, and becomes more pronounced with increasing latitude in the southern hemisphere than in the northern hemisphere. Also the lower stratospheric circulation begins to dominate over the tropospheric circulation as the latitude increases. At high northern latitudes the zonal winds become increasingly irregular due to the variable nature of the stratospheric circulation.

Several major northern hemisphere sudden warmings have been reported during the 10-yr period under consideration, notably in early 1966, 1968, the successive years of 1970 and 1971, and again in early 1973 (e.g. Schoeberl 1978). These phenomena are most important in the high latitude stratosphere above the 10 mb level but as they do sometimes propagate down in altitude and in latitude they could contribute to the excitation function, particularly if the temporary breakdown of the winter circulation occurs at all longitudes. The zonal profile for 80° north does not clearly illustrate the more significant warmings except for the ones in early 1966 and 1970 that appear to have propagated down to the surface. In both examples the westerly circulation broke down in mid-winter and was momentarily replaced by an easterly regime in January–February. The zonally averaged westerly regime re-established itself before being replaced by the end-of-winter reversal. Similar cycles but much less intense and not propagating downwards to the same extent
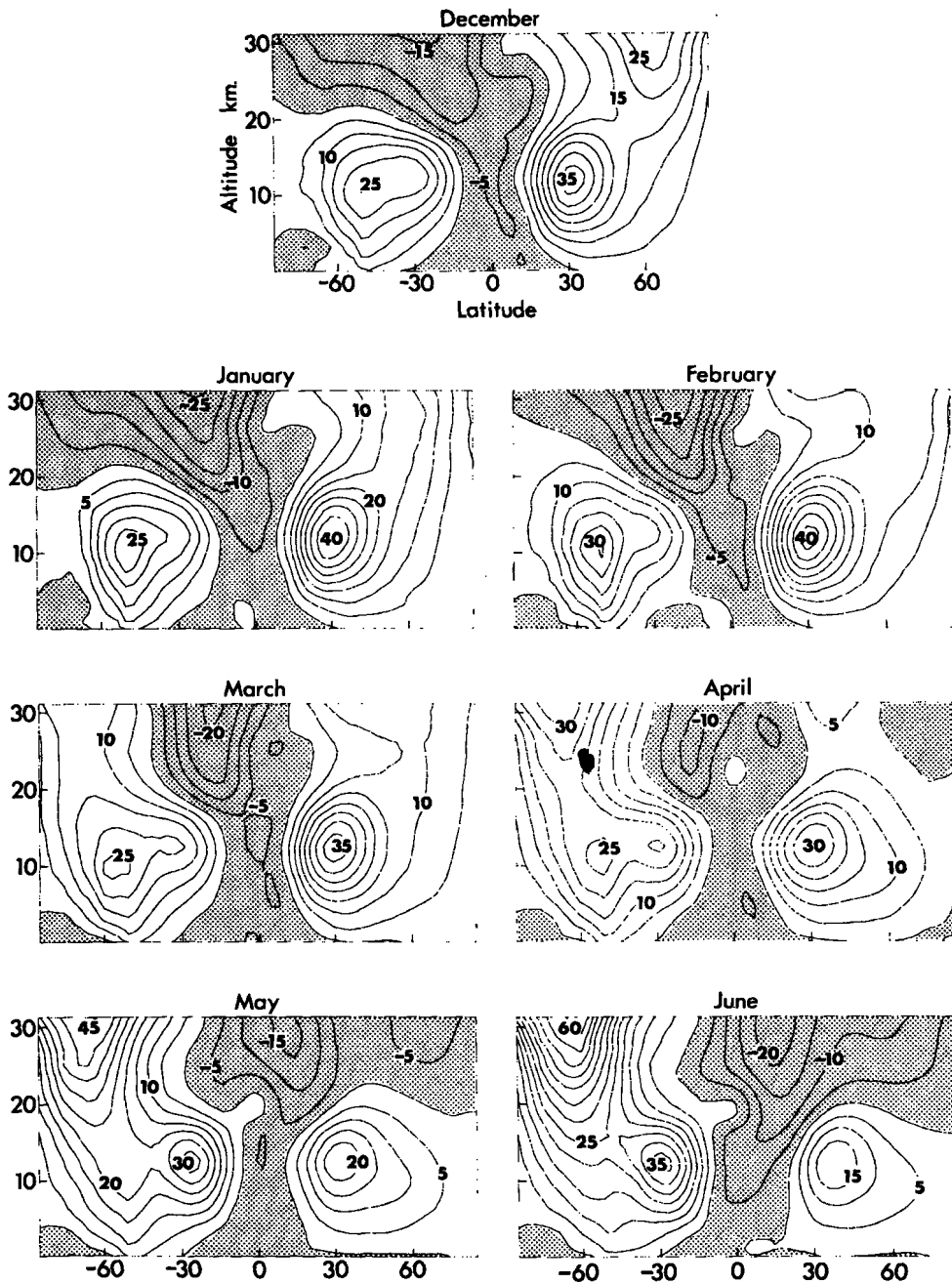
Figure 3. Mean zonal winds as a function of latitude and height for the months from December to June calculated for the 5-yr period from 1963 May to 1968 April. Westerly winds (i.e. blowing to the east) are positive and easterly winds are shaded.

were observed in early 1971 and 1973. The 1968 mid-winter warming does not appear to have been sufficiently intense to cause a reversal of the zonally averaged winds. The zonally averaged winds of the southern high latitudes appear to be largely devoid of these mid-winter reversals.
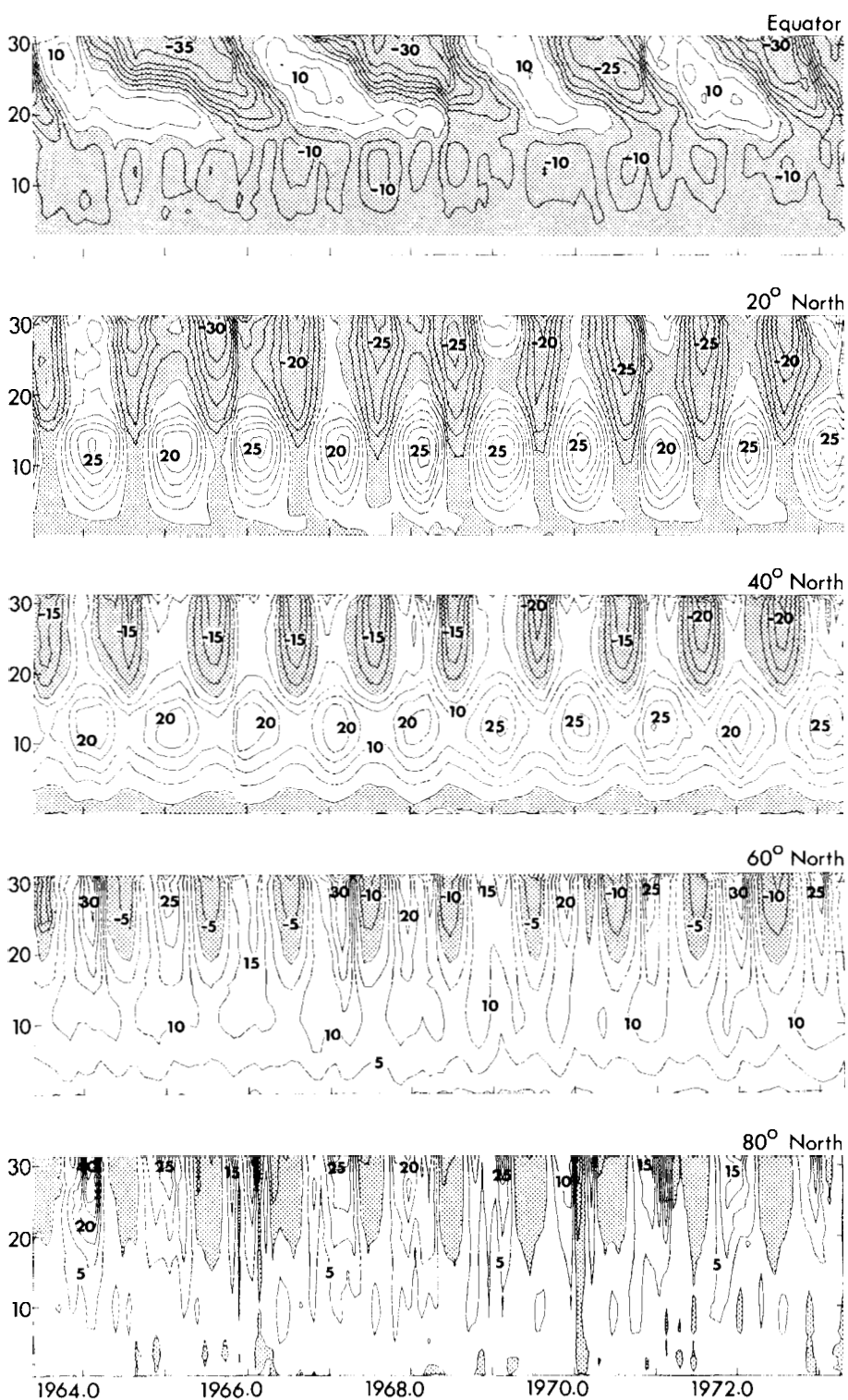
Figure 4. Time–height profiles of the zonal winds along selected latitude bands. The sign convention is the same as in Fig. 3.
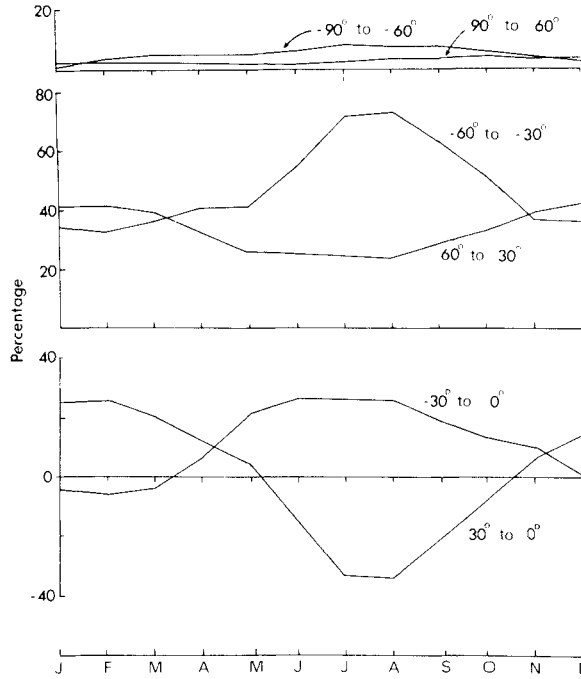
Figure 5. Average percentage monthly contribution to the zonal wind excitation function from winds in 30° latitude bands for the 5-yr period from 1963 May to 1968 April.

Fig. 5 illustrates the average monthly contribution to the excitation function from winds in 30° latitude bands as calculated by equation (9). These results are based on the data from 1968 to 1973 when coverage in the southern hemisphere was relatively complete. The circulations between 60° north or 60° south latitude and the respective poles do not contribute significantly to the average excitation function. More important are the mid-latitudes between 30° and 60°, with the southern hemispheric partial excitation function exceeding that for the same latitude band in the northern hemisphere.

The accuracy σ_ψ of the monthly mean excitation function is difficult to estimate reliably. From equation (9) the variance of the excitation function $d\psi$ for a volume $2\pi r^2 \cos\phi \, dr \, d\phi$ is

$$\sigma_{d\psi}^2 = \left(\frac{2\pi R^3}{\Omega I}\right)^2 \sigma_{u_\lambda}^2 \, \rho^2 \, dr^2 \, \cos^4\phi \, d\phi^2 \tag{10}$$

in which $\sigma_{u_\lambda}^2$ is the variance of the zonally averaged monthly mean wind and represents the combined effect of (i) the uncertainty in the wind measurements, (ii) the spatial variability of the winds within the latitude band and (iii) the temporal variability of the wind during the month over which the measurements have been averaged. Integrating over height and latitude with

$$\sigma_{\bar{u}_\lambda}^2 = \sigma_{u_\lambda}^2 / 30N$$

where $\sigma_{u_\lambda}^2$ is the variance of a single zonal wind measurement, and N is the number of daily wind recording stations in the latitude band $d\phi$, gives, for a typical month

$$\sigma_\psi^2 \simeq 7 \times 10^{-25} \sigma_{u_\lambda}^2, \tag{11}$$

where σ_{u_λ} is in cm s^{-1}. Much of the contribution to equation (11) comes from the southern latitudes, between the equator and $-50°$, where the number of stations in a $10°$ latitude band is typically about 40 compared with more than 100 in the northern latitude band. In Paper II a value of $\sigma_{u_\lambda}^2 \simeq (10 \text{ m s}^{-1})^2$ was adopted for the variance of a single measurement. Then $\sigma_\psi^2 \simeq 0.7 \times 10^{-18}$. Assuming that this variance has equal contributions from all frequencies between f_1 and f_N the error spectrum of ψ will be

$$E_\psi^2(f) = \frac{\sigma_\psi^2}{f_N - f_1}. \tag{12}$$

For f_1 of the order of a fraction of a cycle per year and $f_N = 0.5$ cycle month^{-1}

$$E_\psi^2(f) \simeq 3.5 \times 10^{-12} \qquad \text{s.}$$

If $\sigma_u \simeq 20 \text{ ms}^{-1}$, $\sigma_\psi^2 \simeq 2.8 \times 10^{-18}$ and $E_\psi^2(f) \simeq 1.4 \times 10^{-11}$ s.

5 Further excitation functions

Other seasonal excitations include: (i) atmospheric mass redistribution, (ii) ground water, (iii) sea level, (iv) ocean currents, (v) solid Earth and ocean tides.

The first four excitation functions are discussed by Lambeck (1980). They are small relative to the zonal wind contribution and are also very uncertain. They should therefore be viewed only as orders of magnitude. In particular the sea-level term is very unreliable as it is related to an already uncertain ground-water term through the need to conserve water-mass in the atmosphere, ground and ocean. Only the seasonal variations in these excitation functions can be estimated from the very limited available data (apart from the atmospheric mass term). The most important contribution to the annual term is from the sea-level variation, while the most important contribution to the semi-annual term appears to be the ocean-current excitation associated with the zonal circum-Antarctic current. Table 1

Table 1. Summary of the annual and semi-annual excitation functions and the comparison with m (last line). The results in parentheses correspond to the combined wind, tide and core response only.

	Semi-annual excitation function (P = 6 month)		Annual excitation function (P = 12 month)	
	A	B	A	B
Winds 0–30 km	0.084	0.191	−0.290	−0.194
Winds 30–60 km	0.006	0.048	−0.026	−0.013
Air pressure	−0.001	0.000	−0.003	0.003
Ground water	0.003	−0.005	0.017	0.022
Sea level	0.003	−0.009	−0.011	0.064
Ocean currents	0.052	0.030	(0.02)	
Correction for core response	0.015	0.026	−0.031	−0.012
Tides	0.182	−0.068	−0.032	−0.001
Total	0.344	0.213	−0.376	−0.131
	(0.287)	(0.195)	(−0.380)	(−0.228)
$10^8\psi =$	0.40 cos $[(2\pi/P)t - 28°]$		0.40 cos $[(2\pi/P)t - 199°]$	
	[0.35 cos $[(2\pi/P)t - 34°]$]		[0.44 cos $[(2\pi/P)t - 210°]$]	
$10^8 m =$	0.41 cos $[(2\pi/P)t - 39°]$		0.40 cos $[(2\pi/P)t - 210°]$	

summarizes the results. More important than these fluid contributions is the solid tide at the semi-annual period. With a Love number $k_2 = 0.30$ the solid tide excitation is of a comparable amplitude to the zonal-wind excitation but the latter leads the tide in phase by about $\pi/2$. A value of 0.30 is compatible with the seismic estimates of the Earth's elastic moduli but at low frequencies, due to dispersion effects, the rigidity may be less than indicated by the seismic values (e.g. Anderson *et al.* 1977). This could result in an increase in the Love number k_2 by as much as 4 per cent (Lambeck 1980). Any lag in the solid tide is considered to be very small and inconsequential in the present problem. The ocean tide contribution, estimated on the assumption that the tide follows an equilibrium theory, is about 10 per cent of the solid tide. Also to be considered is the fluid core's response to the tide potential. Merriam (1980) has given an integrated development for the tidal response of an elastic Earth with fluid core and equilibrium ocean tide to obtain $k_2 = 0.315$. This value is used to give the combined tide excitation functions in Table 1. The dispersion effect has been neglected.

In Paper I it was noted that for the semi-annual excitation the stratospheric winds above about 30 km altitude may amount to 20 per cent of the contribution below 30 km, for despite the low densities, the amplitude of the semi-annual winds exceed 30 m s^{-1} over the equator at heights between 40 and 50 km. The contribution of stratospheric winds to the annual term is less important – but still not negligible – since the maximum winds occur both at higher latitudes and altitudes. With a correction for a sign error, the results of Paper I for the excitations from 30 to 60 km are

$$\psi = \left[-0.024 \cos \frac{2\pi t}{12} - 0.010 \sin \frac{2\pi t}{12} + 0.010 \cos \frac{2\pi t}{6} + 0.050 \sin \frac{2\pi t}{6} \right] \times 10^{-8}.$$

From the recent analysis by Belmont, Dartt & Nastrom (1974) of the seasonal stratospheric winds we obtained

$$\psi = \left[-0.026 \cos \frac{2\pi t}{12} - 0.013 \sin \frac{2\pi t}{12} + 0.006 \cos \frac{2\pi t}{6} + 0.048 \sin \frac{2\pi t}{6} \right] \times 10^{-8}.$$

The good agreement between this and the earlier result is mainly due to there being a considerable amount of common wind observations in the two analyses.

6 Comparisons

The 10-yr time-series of monthly values of m and the zonal-wind ψ are given in Fig. 6 and both series indicate pronounced seasonal terms. The most serious defect in the excitation function occurs from 1966 May to October when the southern latitude coverage did not go beyond 40° S and the distortion of the time-series is clearly evident. The corresponding spectra are illustrated in Fig. 7 and these also confirm the existence of the annual and semi-annual terms. The noise spectra (8) and (12) have been superimposed on the power spectra and confirm that there is considerable signal, other than these seasonal terms, that rises above the noise level of both time-series. The phase spectrum indicates essentially a zero lag at the annual frequency but this is not so for the semi-annual term where the rotational response lags the excitation by about 30°–40°. The reason for this is immediately clear from Table 1 where the important tidal contribution is seen to be out of phase with the wind excitation. Fig. 8(a) and (b) illustrate the residual time-series after removal of the two seasonal terms. Both series contain some long-period fluctuations, from about 3–5 yr, as is also suggested by the spectra in Fig. 9 but only between periods of about 16 month and 4 yr are the two in phase. The quasi-biennial term only becomes obvious after some

extensive band pass filtering (Fig. 8c, d). The spectra (Fig. 9) reveal some power that rises above the noise level near 8–9 months where the two series are in phase indicating an atmospheric circulation at these periods, but the coherence between the two time-series at frequencies above about $2 \, \text{cycle} \, \text{yr}^{-1}$ is small, with the possible exception of a terannual cycle.

6.1 SEASONAL FLUCTUATIONS

Table 1 summarizes the amplitude and phase of the annual and semi-annual excitation in the form

$$\psi_\text{P} = \left(A_\text{P} \cos \frac{2\pi t}{P} + B_\text{P} \sin \frac{2\pi t}{P} \right) \times 10^{-8}$$

where P is the period and time is measured from 0 hr January 1. As the observed ψ are average values for 1 month, the coefficients A_P, B_P will be underestimated and a corrective factor has to be applied but only for the semi-annual term is this significant (Jeffreys & Jeffreys 1966, p. 449). Table 1 also summarizes the additional excitation functions discussed in Section 5 as well as the corrective term $(I/I_\text{m} - 1)\psi_*$ which allows for the absence of coupling of the core to the mantle motion, where ψ_* is the total excitation less the tidal contribution for which the core response has already been introduced implicitly through the choice of Love number. The agreement between the total excitation and the astronomically
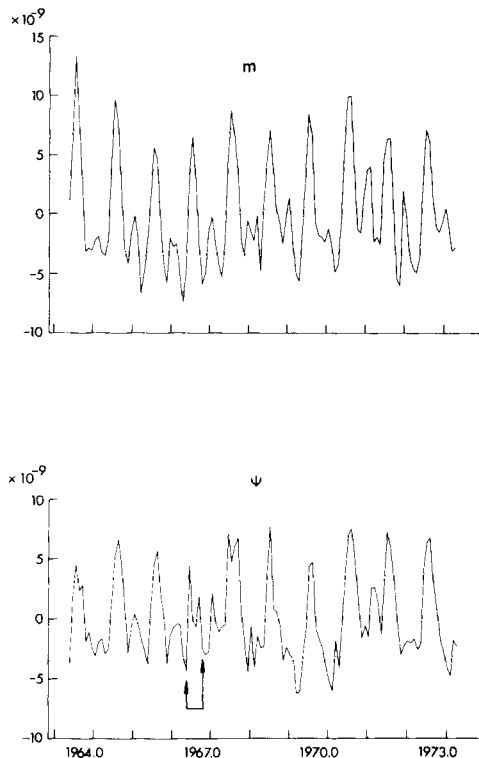
Figure 6. Time-series of the monthly values of the proportional change in length-of-day (m) and of the zonal wind excitation function (ψ) from 1963 May to 1973 April. The interval between the arrows indicates the period when the high latitude southern hemispheric coverage was inadequate.
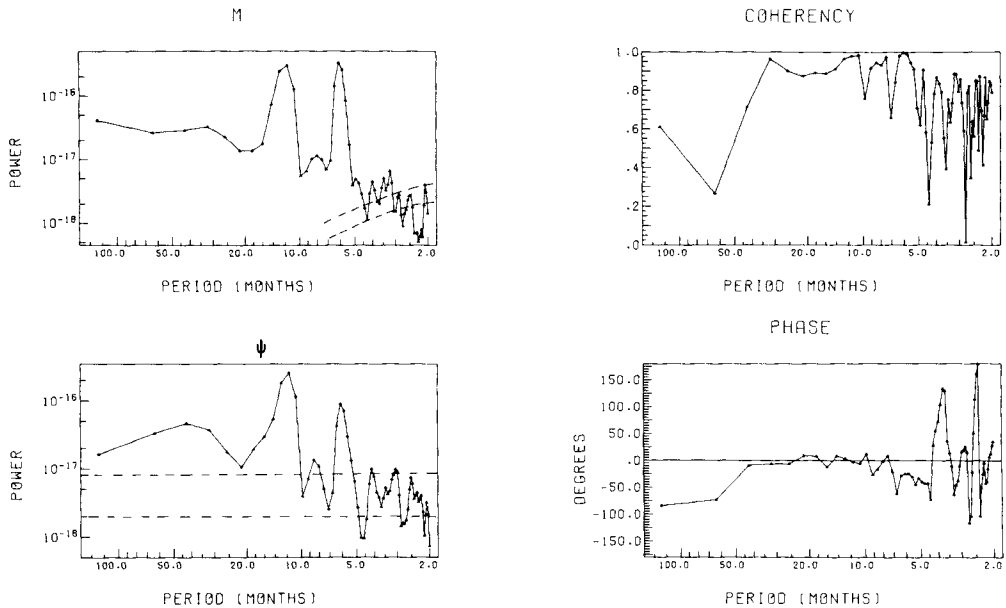
Figure 7. Power spectra for the proportional change in length-of-day (m) and of the zonal wind excitation (ψ). The time-series have been tapered at each end (10 per cent) by a pair of split cosine bell functions and the spectra have been further smoothed by a running mean over three consecutive points with weights 0.25, 0.5, 0.25. The two error spectra in m correspond to $\sigma_T = 1$ ms and 1.5 ms while the two error spectra in ψ correspond to $\sigma_{\bar{u}_\lambda} \simeq 10$ m s^{-1} and 20 m s^{-1}.

deduced values is excellent and we do not even have to resort to Jeffreys' (1916) botanical term to improve upon it! Nevertheless, in view of the uncertainties in the various estimates, the good agreement is probably fortuitous. At both seasonal frequencies the excitation leads the response by about 10° but this is also insignificant in view of the uncertainties in the non-wind and non-tidal excitations: if only the wind and tide excitations are considered together with the appropriate core response the agreement in phase is improved (see the results in parentheses in Table 1).

6.2 THE QUASI-BIENNIAL OSCILLATION

The power spectra (Fig. 9) do not indicate pronounced periodicities near 24 months. Rather, they indicate increased power of 5×10^{-17} for a period range from less than 2 yr to about 4 yr over which the coherency is high and the phase difference is small. This spread in power is a consequence of the erratic behaviour of the quasi-biennial term and of its non-sinusoidal behaviour. In the equatorial regions the angular momentum associated with the zonal winds near the tropopause, for example, can be characterized by a sequence of almost rectangular functions in which the change from easterly momentum to westerly momentum and vice versa occurs over relatively short time intervals between which the amplitudes remain relatively constant. This results in significant second and third harmonics in the power spectra of both m and ψ. In Paper I the amplitude of the quasi-biennial term in m was estimated to be 10^{-9} and this results in a power of the order

$$\tfrac{1}{2} \times 10^{-18} \times 120 \simeq 6 \times 10^{-17} \text{ (month)}$$

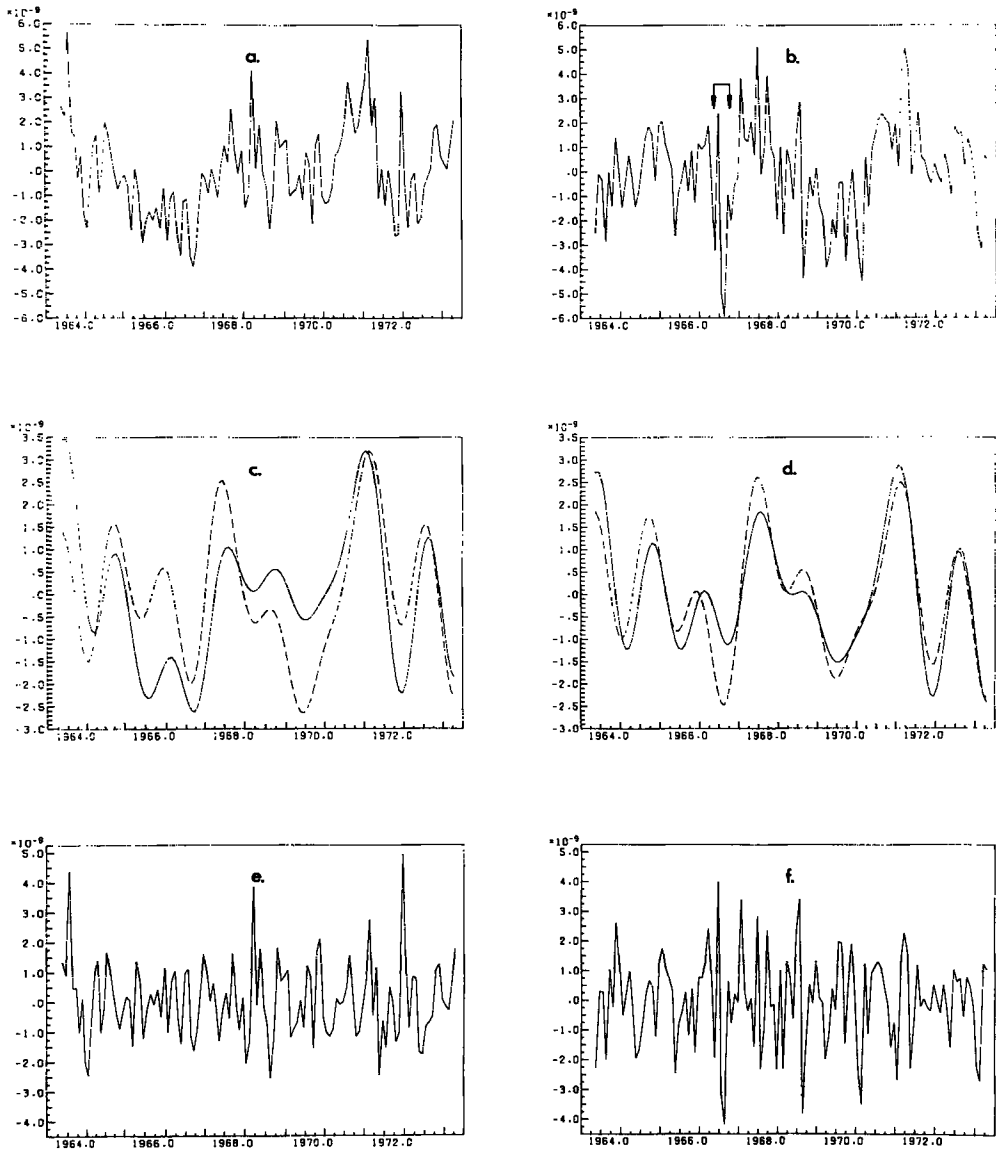for the 10-yr data set, comparable to that observed (Fig. 9).

Figure 8. (a) Time-series of monthly mean m after removal of the 6 and 12 month terms, Δm; (b) time-series of monthly mean ψ after removal of the 6 and 12 month terms, $\Delta\psi$; (c) low-pass filtered time-series of m (solid) and ψ (broken) to pass all periods longer than 15 month; (d) band-pass filtered time-series of m (solid) and ψ (broken) to pass all periods between 15 and 50 month; (e) high-pass filtered time-series of Δm to pass all periods less than 15 month; (f) high-pass filtered time-series of $\Delta\psi$ to pass all periods less than 15 month.

A comparison of m and ψ in the time domain is perhaps instructive. Fig. 8(d) illustrates the variations in m after band-pass filtering so as to retain only periods between 15 and 48 month. The result does not differ very much from that illustrated in Fig. 8(c) in which all periods exceeding 15 month have been retained. There is evidence in this time series of quasi-biennial behaviour with an amplitude that fluctuates in magnitude between 1×10^{-9} and 2×10^{-9}. Of particular note is the variable nature of the period, varying from less than
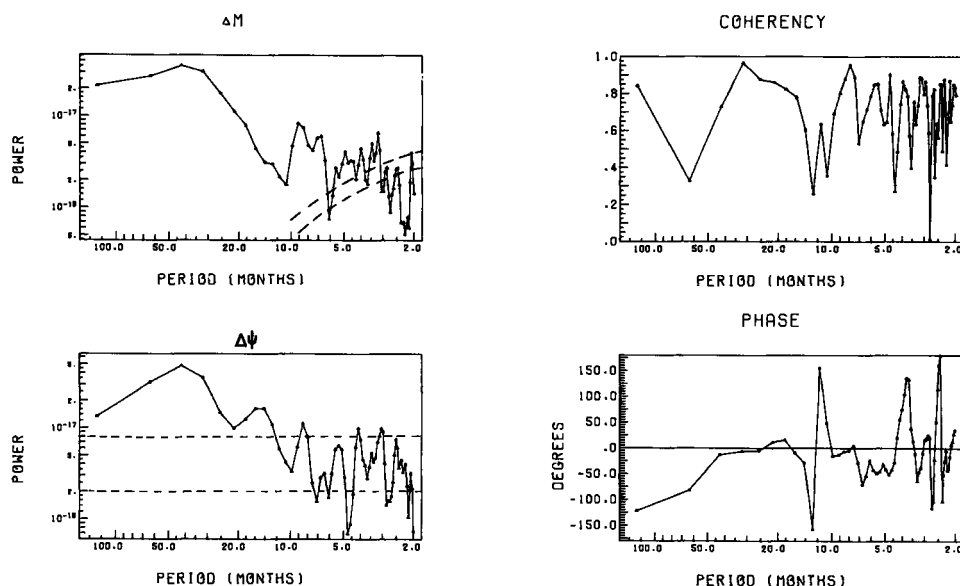
Figure 9. Same as Fig. 7 but after removal of the seasonal terms of 6 and 12 month from the time-series, i.e. Δm and $\Delta \psi$.

2 yr in 1971–72 to more than 3 yr for the preceding cycle. This behaviour is similar to that deduced in Paper I using different filtering techniques. Also illustrated in Fig. 8(d) is the band-pass filtered excitation function. Its behaviour is very similar to that observed in m, both in amplitude and period, confirming the earlier conclusions that most length-of-day variations with periods less than about 4 yr contain a significant contribution from the zonal winds. The principal discrepancy between the two series occurs in mid-1966 and this is a consequence of the incompleteness of the southern hemisphere wind data for a 6-month period. A comparison of the total excitation function with the time–height profiles of Fig. 4 does not reveal a clear relationship with the near equatorial quasi-biennial oscillation because ψ represents an integral effect over all latitudes and heights but, in a general way in the latitude band between $\pm 20°$, times of an accelerating Earth, when $\dot{m}$ is positive, correspond with a change from a predominantly westerly regime to an easterly one.

6.3 LONG PERIOD VARIATIONS

A 'secular' trend occurs in both the ψ and m for the 10-yr period although a casual inspection of Fig. 1 shows that this is probably part of a much longer period fluctuation in lod. For the 10-yr record of m,

$$\dot{m} \simeq -2 \times 10^{-8}/10 \text{ yr}$$

and for the same 10-yr interval in ψ

$$\dot{\psi} \simeq -0.4 \times 10^{-8}/10 \text{ yr}.$$

While the latter is considerably less than the astronomical estimate, it does confirm the earlier observation (Paper II) that the meteorological excitation can contribute up to 10–20 per cent of the total decade fluctuations in lod.

Both the m and ψ spectra (Fig. 9) indicate comparable power at periods greater than about 4 yr although there is more power in m than in ψ. These observations point to an additional, non-zonal wind, excitation function becoming important at periods above about 4 yr. The nature of this additional excitation is perhaps best seen in the time-series of the difference between m and ψ (Fig. 10) and in the spectrum of this difference (Fig. 11). The latter, upon which the error spectra have been superimposed, indicates that most of the power can be attributed to noise, with the exception of that at low frequencies at periods exceeding about 4 yr. The rise in power at about 12 months appears to be mainly a consequence of the inadequate removal of the annual term from the ψ: the variability of the southern latitude station coverage during the first 5-yr period has tended to distort the seasonal pattern in ψ, as is clearly seen in Fig. 6 and in power spectrum of ψ in Fig. 9 (where the smoothing has displaced the peak of the spectrum to a somewhat lower frequency).

The time-series of the 'meteorologically corrected' length-of-day as illustrated in Fig. 10 reveals mainly a long-period term with an amplitude of about 2×10^{-9}. Unfortunately the 10-yr period analysed is devoid of some of the larger and more interesting changes that have occurred in m at other times, such as from 1958 to 1967 when m first increased by 8×10^{-9} in a 4-yr period and then decreased by a similar amount in about 5 yr, or from 1972 to 1978 when m also increased by about 8×10^{-9} (Lambeck 1980). This makes a definitive test of the role of the atmosphere at these longer periods impossible and emphasizes the need to not only go back to the 5-yr data set used in Papers I and II but also to expand the data set beyond 1973. Both of these steps are now in progress.

6.4 HIGH FREQUENCY VARIATIONS

Both the m and ψ spectra in Fig. 9 indicate an increase in power at some periods shorter than 1 yr. The most pronounced signal occurs at 8–9 months at which the phase is also small and the coherency is a maximum. An 8-month periodicity has been noted in near-equatorial zonal winds in the lower stratosphere (Angell & Korshover 1962) and it appears that this may be the third harmonic of the quasi-biennial oscillation. At higher frequencies both m and ψ show some enhancement in power at 4 months although the signal to noise ratio is small. This could be associated with the terannual wave, observed predominantly at mid- and high-latitudes in the stratosphere, and which appears to be the third harmonic of the non-sinusoidal annual term (Belmont *et al.* 1974). The high-latitude height–time profiles of Fig. 4 illustrate this non-sinusoidal seasonal behaviour in that the change from the maximum easterly to maximum westerly wind generally takes longer than the reversal of westerly winds to easterly winds.

The residual time series illustrated in Fig. 8(e) and (f), after they have been band-pass filtered to remove any power at periods greater than 15 months, show considerable month-to-month variability much of which must be considered as noise, for example, the large perturbation in ψ introduced in the latter half of 1966. A careful comparison of the two time-series after mid-1968 does, however, indicate that there is some similarity between the detail in the two time-series in that large excursions in one are generally but not always present in the other.

These correlations can be further investigated with the statistical approach used in Paper II. If the true value of the zonal wind excitation function is denoted by $\hat{\psi}_w$ then

$$\psi_m = \hat{\psi}_w + \delta\psi + \epsilon_m,$$

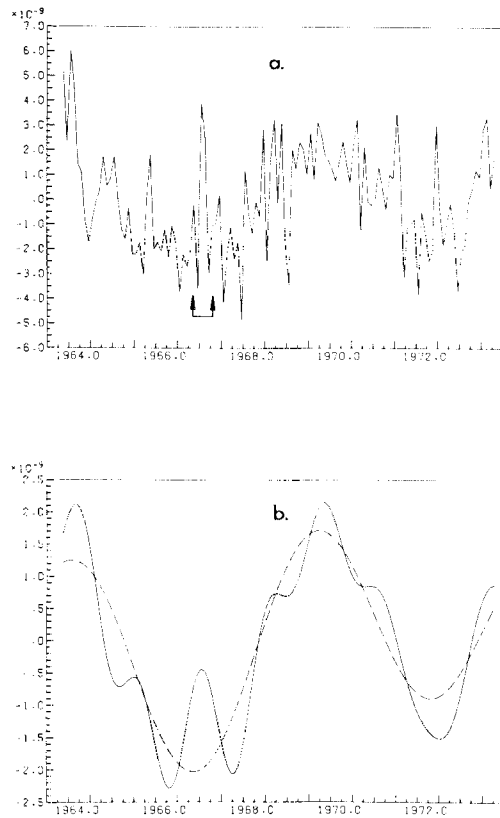where ψ_m is an estimate of the excitation function deduced from the astronomical observa-

Figure 10. (a) The difference between monthly values of Δm and $\Delta \psi$. (b) The low-pass filtered $(\Delta m - \Delta \psi)$ to pass (i) all periods longer than 15 month (solid) and (ii) all periods longer than 50 month (broken).
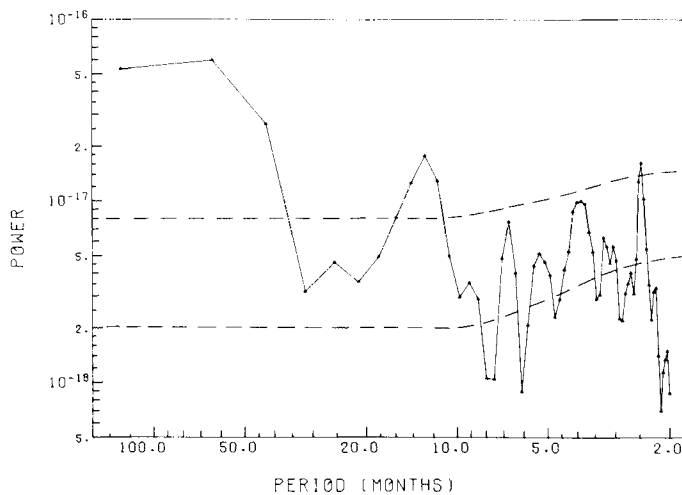

Figure 11. Power spectrum of the difference $\Delta m - \Delta \psi$ with the limits of the error spectrum of this difference.

tions, ϵ_m is the error in the estimate of ψ_m and $\delta\psi$ is any residual excitation. This could be either additional wind contributions, such as the absence of high southern latitude wind data for the early part of the period or the stratospheric circulation that has been systematically neglected in the month-by-month calculation, or it may be some other mechanism not accounted for at all. The zonal wind estimate of the excitation function (previously denoted by ψ) is

$$\psi_w = \hat{\psi}_w + \epsilon_\psi,$$

where ϵ_ψ is the error in the estimate of ψ_w. The four quantities $\hat{\psi}_w$, $\delta\psi$, ϵ_m and ϵ_ψ are independent and it follows that

$$E\{\hat{\psi}_w^2\} = \langle\psi_m\psi_w\rangle, \tag{13a}$$

$$E\{\epsilon_\psi^2\} = \langle\psi_w^2\rangle - \langle\psi_m\psi_w\rangle, \tag{13b}$$

$$E\{\delta\psi^2\} = E\{\psi_m^2\} - E\{\epsilon_m^2\} - E\{\hat{\psi}_w^2\}, \tag{13c}$$

$$E\{(\psi_m - \psi_w)^2\} = \langle(\psi_m - \psi_w)^2\rangle, \tag{13d}$$

$$= \langle\psi_m^2\rangle - 2\langle\psi_m\psi_w\rangle + \langle\psi_w^2\rangle, \tag{13e}$$

$$= E\{\epsilon_m^2\} + E\{\epsilon_\psi^2\} + E\{\delta\psi^2\}. \tag{13f}$$

The $E\{\ \}$ denote the estimates of the mean values and the brackets $\langle\ \rangle$ denote the mean value of the enclosed quantities. The quantity $\langle(\psi_m - \psi_w)^2\rangle$ provides a measure of the agreement between the two estimates ψ_m and ψ_w of the total excitation function, while the quantity $\langle\psi_m\psi_w\rangle$ is a measure of the amount of information common to these two estimates. For perfect astronomical data and for a perfect estimate of the zonal wind excitation,

$$\langle\psi_w^2\rangle = \langle\psi_m\psi_w\rangle \tag{14}$$

even when $\delta\psi$ does not vanish. If the *a priori* variance estimates (8) and (11) are realistic

$$E\{\epsilon_m^2\} = \sigma_m^2, \quad E\{\epsilon_\psi^2\} = \sigma_\psi^2. \tag{15}$$

Table 2 summarizes the statistical results for (i) the total 10-yr record, (ii) for the first 50 months when the coverage for the higher southern hemisphere latitudes was erratic and (iii) for the last 70 months of more satisfactory data. The band-pass filtered series of Fig. 8(e) and (f) were used. An independent estimate of the variance of the zonal wind measurements follows from (13b), (15) and (11). For the total record $\sigma_{u_\lambda} \simeq 16$ m s^{-1} and this suggests that the actual error spectrum $E_\psi^2(f)$ may lie between the two limits indicated in Figs 7, 9 and 11. For the subset of 50 months of data $\sigma_{u_\lambda} \simeq 18$ m s^{-1} and for the second subset $\sigma_{u_\lambda} \simeq 14$ m s^{-1}. With $\sigma_\tau \sim 1$ ms, $\sigma_m^2 \simeq 0.5 \times 10^{-18}$ and with (15) and (13f) this leads to an estimate of the variance of the error of omission $E\{\delta\psi^2\}$ of about 0.6×10^{-18}. If $\sigma_\tau \simeq 1.5$ ms, $\sigma_m^2 \simeq 1.1 \times 10^{-18}$ and $E\{\delta\psi^2\}$ is less than 0.1×10^{-18}. While these results do not rule out the possibility that there are other small excitation functions contributing to the high frequency variations they do confirm the predominant role played by the zonal winds at frequencies above $(15 \text{ month})^{-1}$.

The time-series in Fig. 8(e) and (f) do not indicate any pronounced patterns. There is a suggestion in the astronomical record that the data are noisier than average during the months of March, July and December and, as was suggested in Paper II, this may be attributed to the irregular breakdown of the high latitude winter circulations, but the available data are inadequate to verify this. While some breakdowns, in particular the northern hemispheric
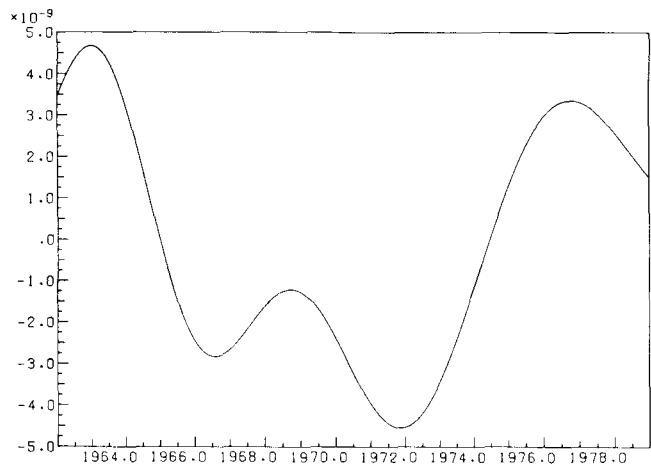
Figure 12. Time-series of residual m for 1962–78 after removal of all zonal wind effects. This represents the combined other-than-zonal-wind excitation, the cause of which is probably related to core–mantle coupling processes.

Table 2. Summary of the statistical tests on the high frequency changes in length-of-day and zonal wind excitation for three periods. $E\{\epsilon_\psi^2\}$ is an estimate of the variance of the monthly mean excitation function and $E\{\epsilon_m^2\}$ is the variance of the monthly values of m. $E\{\delta\psi^2\}$ is a measure of the additional excitation functions not included in the analysis. This quantity has been estimated for two values of $E\{\epsilon_m^2\}$ corresponding to $\sigma_T = 1$ ms and 1.5 ms respectively. In the first case $E\{\delta\psi^2\}$ is about 20 per cent of the mean square value of ψ while in the second case it is less than 5 per cent.

	1963–73 (10^{-18})	1963–67 (10^{-18})	1967–73 (10^{-18})
$\langle \Delta m^2 \rangle$	1.66	1.38	1.86
$\langle \Delta \psi_w^2 \rangle$	2.37	2.56	2.24
$\langle \Delta m \Delta \psi_w \rangle$	0.62	0.31	0.84
$\langle (\Delta m - \Delta \psi_w)^2 \rangle$	2.79	3.32	2.41
$E\{\epsilon_\psi^2\}$	1.75	2.25	1.40
$E\{\epsilon_m^2\}$	0.50	0.50	0.50
$E\{\delta\psi^2\}$	0.54	0.57	0.51
$E\{\epsilon_m^2\}$	1.10	1.10	1.10
$E\{\delta\psi^2\}$	−0.06	−0.03	−0.09

warmings discussed in Section 4, occur at times of rapid changes in m and ψ, not all do. An extension of the day-to-day estimates of the zonal wind excitation function reported by Hide *et al.* (1980) may provide more insight into this question.

7 Conclusions

The analysis of the 10-yr record of zonal wind data and the comparison of the wind excitation function with the astronomical record of changes in the length-of-day, has

confirmed several of the conclusions reached in the earlier papers by Lambeck & Cazenave, namely that; (i) the seasonal terms in length-of-day are predominantly of meteorological origin, (ii) variations in the length-of-day with periods ranging from about 16 month to about 4 yr are also a consequence of the zonal wind circulation and are associated in part with the quasi-biennial oscillation and its harmonics, (iii) at longer periods there is still some power in the zonal wind excitation function. At the decade periods the winds may contribute up to 10–20 per cent of the total excitation, (iv) at high frequencies (> 6 cycle yr^{-1}) the zonal wind appears to be almost entirely responsible for the observed changes in the length-of-day.

One of the primary purposes of studying the relation between the atmospheric circulation and the Earth's rotation is to remove this 'meteorological noise' from the astronomical record and to investigate the residuals for other geophysical excitation functions. The good agreement between the observed and computed annual and semi-annual changes in the length-of-day — both in phase and in amplitude — rules out the possibility that there are other significant but ignored excitation functions. However, about 10 per cent of the total excitations come from hydrological and oceanographic phenomena for which little more than order of magnitude estimates are possible at present. Hence any disagreement between ψ and m cannot be used to draw conclusions about relativistic effects on rotation, as attempted by Rochester & Smylie (1974); about the frequency dependence of the Love numbers; or about departures from the simple model introduced here of the ocean and core response to the surface wind torques. Neither will further improvements in the wind data contribute to the resolution of these questions unless they are also accompanied with a better understanding of the hydrologic and oceanographic contributions.

The present study points clearly to an important zonal wind contribution to the excitation function at frequencies down to about $(4 \ yr)^{-1}$ but as stated earlier, the zonal-wind set must be expanded upon before more definitive conclusions can be drawn. A preliminary attempt at correcting the astronomical data from 1962 to 1978 for the zonal winds is illustrated in Fig. 12 and it is this response that must be attributed to other phenomena, such as core–mantle interactions.

At the very high frequencies the zonal wind excitation is predominant and any other mechanisms will be completely dominated by it unless the winds are known globally and with a better accuracy and resolution than is the case for this 10-yr data set. The question of changes in length-of-day at periods less than a month remains ambiguous and will only be resolved when better high-frequency information on both m and ψ become available.

Acknowledgments

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