Simultaneous inversion of borehole temperature data for determination of ground surface temperature history

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SUMMARY
Recent variations of the ground surface temperature, recorded in the Earth’s subsurface, can be inverted from borehole temperature data. The resolution of the inversion of borehole temperature logs is poor, even when the noise level is low. This report concerns the potential improvement in the resolution of ground surface temperature history when temperature logs from several boreholes are inverted simultaneously. For an inversion algorithm based on singular value decomposition, the singular values obtained for simultaneous inversion of several boreholes do not decrease as rapidly as the singular values for a single borehole. With the same value of the cut-off or of the damping parameter, more eigenvectors remain in the solution and give higher resolution for several boreholes than for one. Tests conducted with synthetic data for 15 boreholes show that the improvement in resolution is real but not spectacular. These tests indicate that borehole temperature logs should have approximately the same sampling interval. Temperature profiles over different depth ranges can be inverted if the reference heat flows and ground temperatures are included in the parameters determined by the inversion. Tests also show that no consistent ground-temperature history (GTH) is obtained when inverting data from boreholes that have experienced different surface-temperature variations. Simultaneous inversion can be applied to: (1) obtain a local GTH for a single site with several boreholes having identical surface conditions, or (2) obtain regional averages with data from different sites that have experienced the same variations in ground temperature.

Temperature profiles measured in four boreholes near Belleterre, in eastern Canada, appear to have recorded the same perturbation. Inversion of the temperature data from individual boreholes yields similar, but not identical, GTHs. The GTH obtained by simultaneous inversion of these four boreholes indicates a cool period, with minimum temperatures at ca. 1800 AD, followed by warming above the reference level. It is consistent with other analyses indicating recent warming in eastern Canada.

Key words: climate, heat flow, inversion.

1 INTRODUCTION
It has long been recognized that past variations of the Earth’s surface temperature are recorded as perturbations to the steady-state temperature conditions of the subsurface. Subsurface temperature is commonly measured in boreholes to determine heat-flow density (HFD). The interest in borehole temperature measurements was recently aroused by the study of Lachenbruch & Marshall (1986) who inferred a 1–2 K warming in Alaska over the past 100 years. They suggested that this warming might be the first sign of an enhanced greenhouse effect predicted by the general circulation models. This interpretation is consistent with the analysis of worldwide temperature records that show strong warming in the Arctic (Hansen & Lebedeff 1987). In the past few years, several studies have been concerned with the inversion of ground-temperature history from borehole data (e.g. Nielsen & Beck 1989; Mareschal & Beltrami 1992; Shen & Beck 1992; Wang 1992).

The temperature logs used for past climate reconstructions are often obtained for heat-flow studies from mining exploration boreholes. Usually, a heat-flow site consists of

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several boreholes within a short (i.e. ≤ 1 km) radius. Because the correlation distance for meteorological trends is larger than 500 km (Hansen & Lebedeff 1987), boreholes from the same region may have been affected by the same surface-temperature trends. If the surface conditions are identical at all the boreholes (which is seldom true), it is likely that the boreholes have recorded similar variations in ground temperature. If this is indeed the case, the subsurface temperature perturbations should be consistent between boreholes, while the errors and noise in the data are incoherent. Stacking temperature perturbations from I different boreholes should improve the signal-to-noise ratio by a factor \( \sqrt{I} \). In practice, the stacking is performed by inverting simultaneously temperature data from all the boreholes in a region. This procedure was applied on a regional scale to boreholes in eastern Canada that have recorded consistent signals (Beltrami & Mareschal 1992). It was also applied locally to all the temperature logs obtained from boreholes in the Oberpfalz in southeastern Germany drilled during studies preliminary to the drilling of the KTB deep hole (Clausner & Mareschal 1995). The purpose of this note is to determine whether this procedure increases the signal-to-noise ratio and improves the stability and resolution of the inversion. Another objective is to examine some of the conditions that the data must satisfy for simultaneous inversion of several temperature logs to be possible. Examples with synthetic and real data illustrate the usefulness and limitations of the procedure.

2 THEORETICAL FRAMEWORK

2.1 Inverse problem

The temperature at depth \( z \), \( T(z) \), is the superposition of the quasi-equilibrium temperature and of \( T_i(z) \), the temperature perturbation caused by the variations in ground surface temperature:

\[
T(z) = T_0 + q_o R(z) - M(z) + T_i(z),
\]

where \( T_0 \) is a reference ground surface temperature, \( q_o \) is the surface heat-flow density, \( M(z) \) is a term due to heat production, and \( R(z) \) is the thermal depth:

\[
R(z) = \int_0^z \frac{dz'}{k(z')},
\]

\[
M(z) = \int_0^z \frac{dz'}{k(z')} \int_0^z A(z') \, dz' = \int_0^z A(z') \, dz',
\]

where \( k(z) \) is the thermal conductivity and \( A(z) \) is the heat production. The thermal conductivity and heat production are usually measured in core samples and/or estimated from the lithology. The two terms \( R(z) \) and \( M(z) \) are thus determined. The inverse problem consists of determining \( T_0 \), \( q_o \), and the ground surface temperature history from \( T(z) \). The inversion procedure is only sketched in this paper, since it has been described in detail in previous papers (e.g. Mareschal & Beltrami 1992).

Since short-period variations are attenuated rapidly with depth, the surface temperature history can be approximated by the average value of the surface temperature during \( K \) intervals:

\[
T_0(t) = T_k; \quad t_{k-1} \leq t \leq t_k \quad k = 1, \ldots, K; \quad t_0 = 0.
\]

The temperature perturbation \( T_i(z) \) is obtained from standard solutions for time-step changes in surface temperature over a homogeneous conductive half-space (e.g. Carslaw & Jaeger 1959):

\[
T_i(z) = \sum_{k=1}^{K} \left( \frac{z}{2\sqrt{\kappa k_{k-1}}} \right) \left( \frac{z}{2\sqrt{\kappa k_k}} \right) \right),
\]

where \( \kappa \) is the thermal diffusivity and \( \text{erfc} \) is the complementary error function.

Eq. (1) can then be written as a linear system of equations:

\[
\Theta_j = A_j X_i,
\]

where \( \Theta_j \) is the measured temperature at depth \( z_j \), corrected for heat production if necessary [i.e. \( \Theta_j = T(z_j) - M(z_j) \)]. \( X \) is a vector containing the \( K + 2 \) unknowns, \( T_0, q_o, T_1, \ldots, T_k \). The matrix \( A_j \) contains 1 in the first column, the thermal resistance to depth \( z_j \), \( R(z_j) \), in the second column, and the differences between complementary error functions at times \( t_{k-1} \) and \( t_k \) for depth \( z_j \) in columns 3 to \( K + 2 \).

2.2 Simultaneous inversion

For I boreholes, the unknown parameters are the I surface temperatures and HDFVs and the K parameters of the ground-temperature history. The data are all the temperature measurements from all the boreholes. If \( N_i \) is the number of temperature measurements at borehole \( i \), the matrix has \( N_1 + N_2 + \ldots + N_I \) rows and \( K + 2 \) columns. The first \( N_i \) elements of the first column equal 1 and all the others 0, the following \( N_i \) elements in the second column are 1 and all the other elements are 0, and so on. The following I columns contain the thermal resistances to depth \( z_j \) in borehole \( i \). Finally, the K last rows contain the differences between error functions at times \( t_k \) and \( t_{k-1} \) for every depth and every borehole:

\[
\begin{bmatrix}
\theta^{(1)} \\
\theta^{(2)} \\
\vdots \\
\theta^{(I)}
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & 0 & 0 & R^{(1)} & 0 & 0 & 0 & A^{(1)} \\
0 & 1 & 0 & 0 & R^{(2)} & 0 & 0 & A^{(2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 1 & 0 & 0 & R^{(I)} & A^{(I)}
\end{bmatrix}
\times
\begin{bmatrix}
T_0^{(1)} \\
T_0^{(2)} \\
\vdots \\
T_0^{(I)} \\
q_0^{(1)} \\
q_0^{(2)} \\
\vdots \\
q_0^{(I)} \\
T
\end{bmatrix},
\]

where \( \theta^{(i)} \) denote the vectors of temperature data for borehole \( i \), \( R^{(i)} \) denote the vectors containing thermal resistance to each depth in borehole \( i \), the elements of the matrix \( A^{(i)} \) are the differences between error functions at time \( t_k \) and \( t_{k-1} \) for each depth of borehole \( i \). The unknown parameters are the I reference surface temperatures \( T_0^{(i)} \), the I reference HDFVs \( q_0^{(i)} \),
and the K parameters of the ground-temperature history contained in the vector T.

2.3 Singular value decomposition

The system of N linear equations (7) must be solved for the M = K + 2I unknowns. In general, this system is mixed-determined, i.e. over- and underdetermined. It is also unstable. Such systems are commonly solved by singular value decomposition (SVD) (e.g. Lanczos 1961; Jackson 1972; Menke 1989). The SVD is based on the following decomposition of the (N x M) matrix A:

\[ A = U \Sigma V^T \]  

where A is an (N x M) diagonal matrix whose elements are the non-zero 'singular values' \( \lambda_i \), (i = 1, ..., L), where \( L \leq \min(M, N) \), U is an (N x N) orthonormal matrix of eigenvectors spanning the data space and \( V \) is an (M x M) orthonormal matrix of eigenvectors spanning the model space, and subscript T indicates the transpose. If \( L \leq M \), the system of equations is underdetermined; if \( L \leq N \), it is overdetermined. When the system of equations \( Ax = b \) is mixed-determined, it admits a generalized solution, given by:

\[ x = V \Sigma^{-1} U^T b, \]  

where \( \Sigma^{-1} \) is an \( M \times N \) diagonal matrix whose elements are \( 1/\lambda_i \). Eq. (9) shows that the very small singular values make the solution unstable to small perturbations in the data (i.e. noise and errors). In practice, this problem is alleviated by retaining in the solution only the \( P \leq L \) singular values that are larger than a cut-off value. Although the solution obtained in this way is no longer the 'true' solution, it contains the main features of the solution (Menke 1989; Parker 1994). This solution is often not very smooth. A smooth and stable solution can be obtained by adding a small constant to each singular value:

\[ \lambda_i \rightarrow \lambda_i^2 + \epsilon^2 \]  

This does not affect the inverse of the larger singular values but stabilizes the inversion by damping to 0 the inverse of the smaller singular values. In several previous papers (Mareschal & Beltrami 1992; Beltrami & Mareschal 1992), we have chosen to use a sharp singular-value cut-off, where only a few eigenvectors are included in the solution. The solution so obtained often contains oscillations (similar to the Gibbs effect in Fourier series expansion). Tests with noise-free synthetic data show that these oscillations decrease when more eigenvectors are retained. By damping the smaller eigenvalues, we obtain a smoother solution that appears 'nicer' (but is by no means better). Although there is no compelling argument for either procedure, we have chosen to damp the smaller eigenvalues and reduce these oscillations, which are an artefact of the procedure. Tests with synthetic and real data were performed with both procedures. To avoid repetitions and keep the length of the paper within reasonable limits, only the results obtained with damped inverse singular values will be presented.

The model resolution matrix \( Q \) is defined as (Menke 1989):

\[ Q = V I V^T, \]  

where \( I \) is a diagonal matrix with elements \( \lambda_i^2/\lambda_i^2 + \epsilon^2 \). The resolution matrix \( Q \) depends only on \( \epsilon \) and on the data kernel of \( A \); it is independent of the values of the data. The meaning of \( Q \) can be understood by writing (e.g. Menke 1989) \( T^{est} = QT^{true} \). If \( I = I \), the resolution matrix is the identity matrix: the model parameters are perfectly resolved. If \( Q \neq I \), the model parameters are averages of the 'true' model parameters. The damping of the small eigenvalues stabilizes the solution at the expense of the resolution.

3 INVERSION OF SYNTHETIC DATA

3.1 Impact of noise in the data

To illustrate how noise in the data affects the inversion procedure, 15 synthetic temperature-depth profiles were generated. This number is a practical limit, because of the limited number of suitable borehole temperature data available. The boreholes are assumed to be 600 m deep and sampled at 10 m intervals. The distribution of the singular values and the model resolution matrix depend only on the sampling of the data and of the parametrization of the GTH. For all the examples, the parametrization assumes a ground surface temperature history of 1000 yr, averaged over 100 intervals of 10 yr. For deeper boreholes that have recorded a much longer GTH, a different parametrization with a logarithmic distribution of time intervals would reduce the number of parameters and be advantageous (e.g. Mareschal & Vasseur 1992). For the examples in this study, a logarithmic distribution is not necessary.

The singular-value spectra for inversion of one and 15 boreholes are compared in Fig. 1. The largest singular value corresponds to a vector in model space whose largest component is the reference heat-flow density. The vector corresponding to the second-largest singular value has its largest component in the reference surface temperature. The other singular values correspond to a superposition of GTH parameters. For 15 boreholes, the first 15 singular values are identical and the corresponding vectors are linear combinations of HFDs. The next 15 values correspond to vectors with a very large component in the surface temperatures. The singular-

Figure 1. Singular-value spectra for inversion of one and 15 temperature logs. For 15 logs, the first 15 and the next 15 singular values are superposed. The following singular values are larger for 15 logs than for one.
value spectra show that the reference temperature and heat flow are well resolved and the other parameters are not. Because the singular values decrease more rapidly for one borehole than for 15, for the same value of $\varepsilon$, the resolution of 15 boreholes will be better. This improvement can be seen more directly in the model resolution matrix.

The model parameters can be determined directly by multiplying the 'true' parameters (the reference temperatures and gradients and the GTH used for the forward problem) by the model resolution matrix. The model resolution matrices for single and simultaneous inversion with $\varepsilon = 0.005$ are compared in Fig. 2. The model resolution matrices show that some parameters (reference temperature and heat flow) are almost perfectly resolved. For the other parameters, the resolution is poor. For the same value of the damping factor, $\varepsilon$, there is some improvement for simultaneous inversion, but this improvement is not spectacular and resolution remains low for the early part of the GTH. In principle, multiplication of a given GTH by the resolution matrix predicts what would be obtained from inversion. This, however, does not include the impact of the noise.

A temperature history consisting of a 2.5 K linear decrease in temperature between 1500 AD and 1750 AD, followed by a 4.5 K increase between 1750 and present was assumed to calculate the transient perturbation. 15 temperature logs were generated by superposition of this perturbation to a reference temperature profile with surface temperatures between 0 and 5 °C and a gradient between 0.01 and 0.015 K m$^{-1}$. Normally distributed random noise of a variable level was added to the synthetic temperature logs (Press et al. 1992). In the following, the noise level refers to one standard deviation.

Fig. 3 compares the ground-temperature history inverted from a single borehole (a) with that of 15 boreholes (b) for essentially noise-free (5 mK noise level) and noisy (500 mK noise level) data. The latter value is much higher than the noise due to measurement errors or to the neglect of small changes in thermal conductivity. In the noiseless case, the model parameters are relatively well resolved. For low noise level, the inversion results show a relatively good model resolution when compared with the 'true' GTH. Because the numerical noise is higher for 15 boreholes, the value retained for $\varepsilon$ is higher. As expected, when the noise level increases, the difference between the estimated and 'true' GTHs is large because the smaller singular values have been damped. The resolution is low for inversion of a single log ($\varepsilon = 0.01$); in particular, the minimum around 1750 AD has been damped. This minimum is much better resolved with simultaneous inversion, although the value of $\varepsilon$ is larger (0.02).

In general, the noise level is unknown and may vary between available temperature–depth profiles. To estimate the effect of variable noise levels in individual profiles, temperature logs were generated with a surface temperature set at 3 °C and an
equilibrium gradient set at 0.01 K m\(^{-1}\), and the same GTH as that used for the previous example. Variable noise between 2.5 and 250 mK was added to obtain 15 temperature–depth profiles. Temperature logs were inverted individually before being inverted simultaneously. Table 1 shows how the value of the damping parameter used in the inversion of individual profiles must be increased because of noise. The results of inversion of individual profiles displayed in Fig. 4(a) demonstrate the decrease in resolution of the GTHs with increasing noise level. Fig. 4(b) shows the GTH obtained from the simultaneous inversion of all 15 noisy temperature–depth profiles. With \( \varepsilon = 0.02 \), the resolution of the simultaneous inversion is comparable to the inversion of a single log with the lowest noise level. The values of \( \varepsilon \) given in Table 1 cover the range encountered in most practical applications (0.01 \( \leq \varepsilon \leq 0.4 \)).

### 3.2 Sampling interval and simultaneous inversion

Not all the available borehole temperature logs were sampled at the same interval (some older logs are in non-metric units). In order to evaluate the weighting placed by the sampling rate on the GTH obtained by simultaneous inversion, synthetic temperature logs were generated from the same GTH as in the previous example (GTH-1) and one opposite in sign (GTH-2). A noiseless temperature–depth profile generated from GTH-1 was sampled at 5, 10 and 20 m from the surface down to 600 m. The temperature–depth profile generated from GTH-2 was sampled every 10 m over the same depth range.

The solutions obtained from the simultaneous inversion of the temperature log from GTH-1 with each of the logs from GTH-2 are shown in Fig. 5(a). The same parametrization and \( \varepsilon = 0.05 \) were used throughout these inversions. When the sampling rate is the same for both profiles, the inversion yields a null solution. In the case of different sampling rates, the GTHs obtained are not close to either GTH-1 or GTH-2. The very recent part of the GTH is characterized by a strong oscillation of short period (4 K peak to peak over 10 yr). However, there seems to be a trend in the low-frequency part that is biased towards the temperature history from the profile with the higher sampling rate. Increasing the value of \( \varepsilon \) until the oscillation is eliminated yields a GTH that is quasi-null (Fig. 5b). These results indicate that although the solution is weighted towards the temperature profile with the higher sampling rate, it is likely that, if the temperature logs have recorded conflicting GTHs, the inversion will not prove feasible and will not yield a false GTH. Simultaneous inversion is not equivalent to averaging the GTHs obtained from each log, nor to the inversion of the averaged temperature perturbation evaluated over the same depth range. These results suggest that simultaneous inversion should be carried out only for data with approximately equal sampling rates to avoid bias.

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Ground-temperature history obtained by inversion of data with variable noise levels between 2.5 and 250 mK. (a) Inversions from single logs with 2.5, 25 and 250 mK noise. (b) Inversion from 15 logs (five with 2.5 mK, five with 25 mK and five with 250 mK noise). The value of \( \varepsilon = 0.02 \) is the same as that for 15 logs with 2.5 mK noise.

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Ground-temperature history obtained from inversion of noiseless data with different sampling rates. The synthetic profiles assume two opposite ground-temperature histories. When the sampling rate is the same, a null result is obtained. (a) With different sampling rates, the inversion yields high-frequency oscillations for \( \varepsilon = 0.05 \). (b) For \( \varepsilon = 0.5 \), the oscillations disappear, but most of the GTH has been damped as well.
If there is a large sampling difference (i.e. a factor of 2), interpolating temperature between data from the undersampled log would give the same weight to the two logs; alternatively, decimating the oversampled log puts the same weight on the two data sets. Except for continuous temperature logs, data are taken with similar sampling rates, except for differences caused by the dip of some boreholes. This is the case for all the data collected in eastern and central Canada during the past ten years, where interpolation is not needed.

3.3 Temperature-depth profiles of different lengths

Temperature profiles of different lengths contain a record of ground surface temperature changes over different time spans. Does joint inversion of profiles of variable lengths recover a consistent GTH?

Synthetic temperature logs were generated from GTH-1 that were sampled every 10 m from surface to 300 and 600 m. The reference surface temperatures and gradients were set at 3 °C and 10 mK m⁻¹ respectively.

Fig 6 compares the GTHs obtained by inversion of individual temperature profiles with the GTH obtained by the joint inversion of both temperature logs. The solution for the joint inversion is almost identical to the GTH from the individual inversion of the deeper temperature profile. The differences between these GTHs and that obtained from the 300 m log mostly concern the period prior to 1800 AD, for which the shorter temperature profile lacks information. Table 2 shows the reference surface temperatures and gradients calculated for each profile by single and joint inversion. The reference surface temperature calculated by the inversion of the 300 m log alone is 0.5 °C lower than its true value, while the gradient is 1.57 mK m⁻¹ (i.e. 15 per cent) higher than its true value. The difference in the most recent part of the GTH that appears in Fig 6 is an artefact because the reference surface temperature calculated for the 300 m borehole is 0.5° lower than that of the 600 m borehole. Thus, if the actual ground temperature was displayed, rather than its departure from the reference level, the differences between the GTHs would be attenuated after 1800 AD and increased before 1800 AD.

It is noteworthy that the reference surface temperature and gradient for the 300 m log obtained from joint inversion were close to their true values, while the gradient obtained from single log inversion was 15 per cent higher than its true value. Simultaneous inversion of shallow and deep boreholes might not yield more information on the GTH than inversion of the deep boreholes, but it could improve the estimates of heat-flow density from the shallow boreholes.

The results of the same tests with noisy temperature profiles are given in Fig 6(b). In this case, a larger value of ε was used. The inversion of both profiles gives almost the same GTH as the inversion of the 600 m profile. Noise decreases the resolution, but the general features of the synthetic GTH are still recovered.

4 EXAMPLE WITH REAL DATA

Four mining-exploration boreholes drilled in 1985 and logged for temperature in 1988 (Pinet et al. 1991) near the town of Belleterre (78°43W, 47°24N; Québec, Canada) are used to illustrate this discussion (Fig. 7). These four boreholes are within a 150 m x 150 m square (see Table 3 for information on site location). The vegetation consists of short pine trees and does not vary much, providing similar solar exposure conditions at all sites. From the surveyor’s map, the maximum difference in surface elevation between boreholes is 4.7 m. The overburden is thin (<5 m) and the main rock types are interlayered intermediate to mafic volcanics. The thermal conductivity was measured on rock samples: it varies between 2.8 and 3.8 W m⁻¹ K⁻¹ and shows no systematic trend within each borehole. For some samples of basalt, an anisotropy ratio of 1.4 was found (Pinet et al. 1991). We have not investigated the effect of conductivity anisotropy on the interpretation. Differences in average conductivity between boreholes are 10 – 15 per cent (Table 3). Because no systematic trend in conductivity was observed, the thermal conductivity was assumed constant within each borehole. The temperature measurements were made at 10 m intervals in all boreholes, but because the boreholes are dipping at variable angles, the sampling depths are not exactly equal. Also, the dip decreases with depth in all the boreholes. The dip angle was previously measured with the standard acid-test procedure. The temperature was logged from 20 m to a maximum depth between 358 and 420 m (see Table 4). The differences in sampling
interval and depth of the boreholes are sufficiently small to have little impact on the inversion. The temperature profiles are shown on Fig. 8. Two values seem markedly anomalous (at ~200 m on 88-11 and ~100 m on 88-12), and are possibly measurement errors. The cause for the variations in slope around 200 m on profiles 88-12 and 88-13 is not certain, but water flow has been suggested.

The temperature logs were inverted individually for ground-temperature parameters consisting of 100 intervals of 10 yr. The values of \( \varepsilon \) used and the reference surface temperature and gradients obtained in each inversion are listed in Table 4. The reference surface temperatures are not very different. The individual GTHs (Fig. 9) agree well in terms of the general warming trend between 1800 AD and the present, but they differ in the amplitude of the warming (between 2 and 3 K) and the details of the very recent (last 20 yr) history. The recent history is not so well constrained because no data were recorded above 20 m. Only one borehole indicates prior cooling with the minimum ground temperature reached ca. 1800 AD.

In order to determine better the amplitude of warming, it is necessary to verify whether there exists a single GTH that fits the four temperature profiles simultaneously. This is reasonable
since it is expected that the climatic forcing has been the same throughout this area (<1 km²). The four temperature logs were inverted simultaneously with the same parametrization of the ground-temperature history as was used for the inversion of individual logs. The result is also shown in Fig. 9. The values of the surface temperature and equilibrium or reference gradient for individual and simultaneous inversion for each temperature log are given in Table 4. The reference surface temperature is about the same for all sites as one would expect because of the similar surface conditions.

The GTH from simultaneous inversion can be considered as an average history of the ground-temperature changes since it fits the four temperature logs included in the analysis. This GTH contains a cooling trend prior to 1800 AD followed by 2.5 K warming ending in 1940. The cooling, which appears in the inversion of only one of the temperature profiles, is not well marked. This is due in part to the relatively shallow depth of these boreholes.

5 CONCLUSIONS

Tests were conducted with synthetic data to determine whether the resolution of inversion is improved by simultaneous inversion of several temperature profiles. These tests showed the following.

1. When inverting a single temperature log, the value of the damping parameter $\xi$ (and thus the resolution) is determined by the noise level. Simultaneous inversion of several temperature logs results in a marginal improvement of the resolution determined by the data set with the lowest noise level.

2. Simultaneous inversion of several temperature profiles must be carried out only for temperature logs with approximately the same sampling rate.

3. It is possible to invert simultaneously temperature profiles with different depths, provided that the reference HFDs and surface temperatures are also inverted. It is not possible to invert the perturbations calculated as the difference between the measured temperature and the reference temperature profiles obtained by upward continuation of the lower part of the profile.

4. Simultaneous inversion of data from several boreholes that have recorded inconsistent signals is likely not to yield a false GTH.

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